2nd International Conference on the Material Point Method for Modelling Soil-Water-Structure Interaction

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Paper number: mpm2019.85
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# SPH study of water wave interaction with solid/porous boundaries

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## ABSTRACT

In this paper, a SPH method is presented for studying the interaction between water and porous structures. Porous particles are deployed in the background to facilitate the determination of the permeability of the structures and the modification of the apparent density of the particles inside the porous material. The proposed model gives the results that are in a good agreement with the past experimental results. The model has been applied to study the solitary wave propagation through a porous block located on a sloped beach. The results indicate the present model's good capability for simulating the water flow past various porous materials including soils, wave absorbing blocks and coastal vegetation.

**KEY WORDS:** SPH; two-phase flow; porous structure; dambreak; solitary wave; run-up

## INTRODUCTION

Wave interaction with porous media is of great interest in coastal engineering as it is widely observed around coastal structures such as breakwaters, coastal dykes and coastal forests. Those structures protect coastal areas from wave attacks. They are often regarded to be more effective than the solid impermeable structures in defending the coastlines. However, they sometimes fail to sufficiently dissipate wave energy, leading to overtopping, scouring and erosion of seabed (Oumeraci, 1994). Therefore, the detailed analysis of the wave motion around porous structure and the accurate evaluation of the wave impact are required for more effective coastal protections.

Smoothed Particle Hydrodynamics (SPH) was originally developed for astrophysical problems (Lucy, 1977). Since Monaghan (1994) applied SPH to free surface flows, SPH has been widely adopted in the field of computational fluid dynamics. It is found suitable for simulating large deformation of the free surface, which is important in studying breaking waves and wave impacts. The original SPH method adopts the weakly compressible assumption. Cummins and Rudman (1999) and Shao and Lo (2003) developed incompressible SPH (ISPH) which enforces incompressibility by a fractional time scheme and by solving Poisson Pressure Equation (PPE). Shao (2010) first applied ISPH to the simulation of water interaction with porous media. In his model, the porous effect is represented by additional friction force terms in the Navier-Stokes (NS) equations. However, Shao (2010) assumed that the turbulent flow effect could be neglected similar to Liu et al. (1999), and used grid lines at the interface between the porous and pure fluid regions. This is not a proper method since SPH is a mesh-free method and particles move continuously through the porous region into the fluid region and vice versa. Many studies have attempted to propose a better treatment of the interface and to improve the porous flow model (e.g. Akbari 2014; Gui et al. 2015a; Pahar and Dhar 2017; Khayyer et al. 2018). This paper presents a general ISPH model for investigating the water wave interaction with porous structures and demonstrates an example concerning the solitary wave run-up on a sloped beach occupied by a porous block.

## METHODOLOGY

Due to the limitation of pages, only brief summary is explained in this paper.

### Governing Equations

In SPH, the continuity and momentum equations are represented in the Lagrangian form in the pure fluid region.

For the porous flow, following Peng et al. (2017) and Khayyer et al. (2018), the equations can be written as:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g} + \frac{\boldsymbol{R}}{\rho} \tag{2}$$

where $\boldsymbol{u}$ is the velocity of a particle, $\rho$ is density, $P$ is pressure, $\nu$ is kinematic viscosity, $\boldsymbol{g}$ is a gravitational acceleration vector and $\boldsymbol{R}$ is resistance force due to porous media. The resistance force $\boldsymbol{R}$ is expressed in the following equations (Peng et al. (2017)):

$$\boldsymbol{R} = -\frac{\mu}{Kp}\boldsymbol{u_d} - \frac{1.75}{\sqrt{150}}\frac{\rho}{\sqrt{Kp}Nw^{\frac{3}{2}}}|\boldsymbol{u_d}|\boldsymbol{u_d} \tag{3}$$

$$Kp = \frac{Nw^3 Dc^2}{150(1 - Nw)^2} \tag{4}$$

Where $\mu$ is dynamic viscosity, $Kp$ is permeability, $Nw$ is the porosity of porous material, $\boldsymbol{u_d}$ represents Darcy velocity and $Dc$ is a mean grain diameter of porous media.

## Modified Source Term of PPE

In ISPH, the velocity at the next time step $k$+1 can be projected by the summation of an intermediate velocity $\boldsymbol{u}$* and a corrected velocity $\Delta\boldsymbol{u}$. From Equation (2), the velocity at the intermediate time step can be calculated except pressure term as follows:

$$\boldsymbol{u}^* = \boldsymbol{u}^k + (\nu\nabla^2\boldsymbol{u} + \boldsymbol{g} + \frac{\boldsymbol{R}}{\rho})\Delta t \tag{5}$$

where $\Delta t$ represents an increment of time.
In the correction step, the velocity increment is calculated by pressure field.

$$\Delta\boldsymbol{u} = -\frac{1}{\rho}\nabla P^{k+1}\Delta t \tag{6}$$

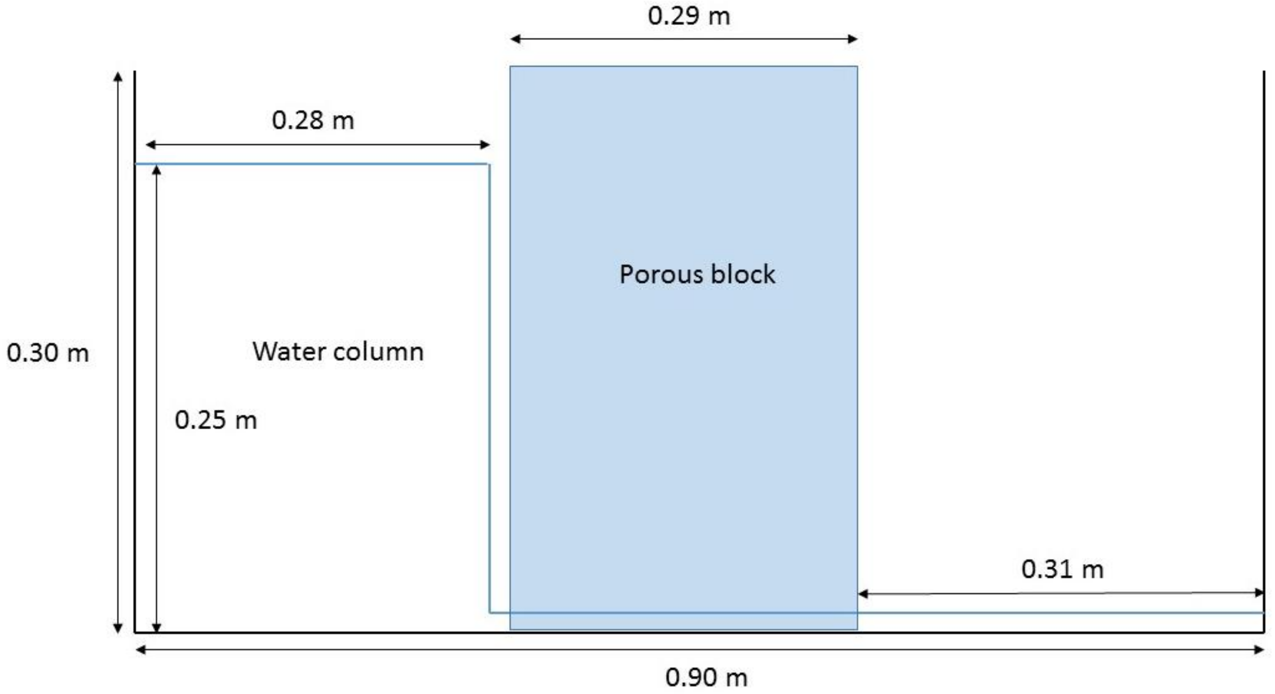
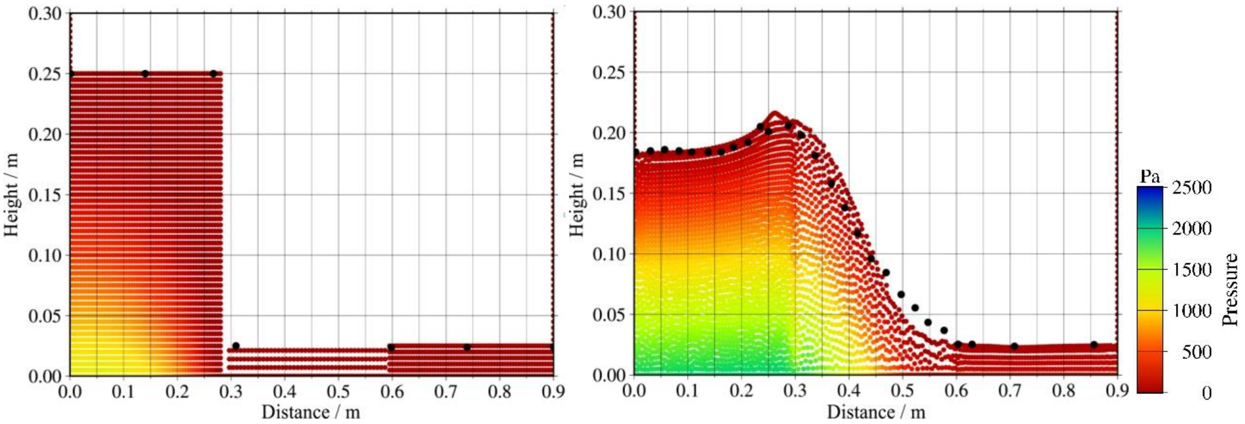


Figure 1 Numerical set up of dam breaking simulation with porous block

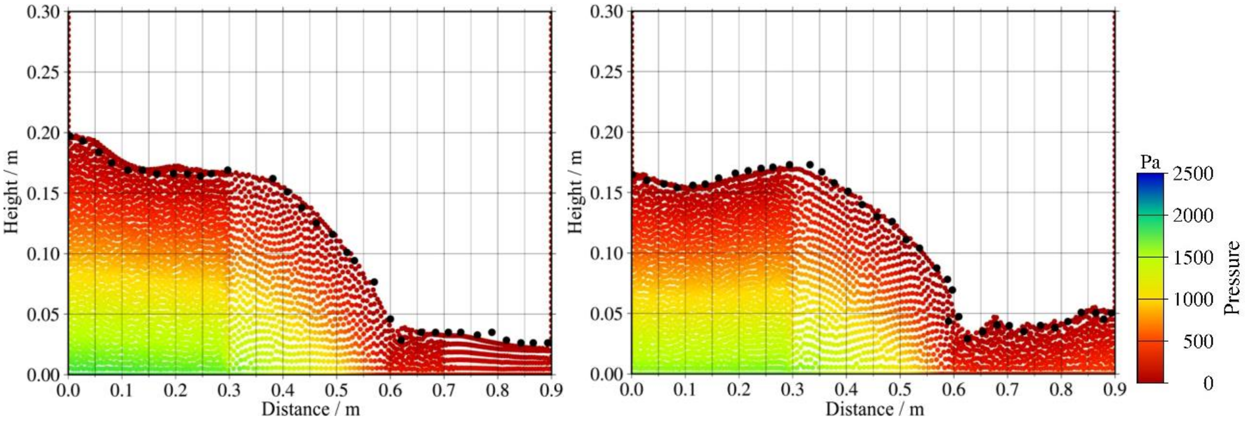
The pressure field at the next time step can be obtained by solving PPE. In this paper, the source term of PPE is based on High-order Source (HS) scheme (Khayyer and Gotoh (2009)) and its combination with a standard source term (Guiet al.(2015b)) in the following form:

$$\nabla \cdot \left( \frac{1}{\rho^*} \nabla P^{k+1} \right) = \theta \frac{1}{\rho_0 \Delta t} \left( \sum_j m_j \nabla_i W_{ij} \cdot \boldsymbol{u}_{ij} \right)^* + (1-\theta) \frac{\rho_0 - \rho^*}{\rho_0 \Delta t^2} \tag{7}$$

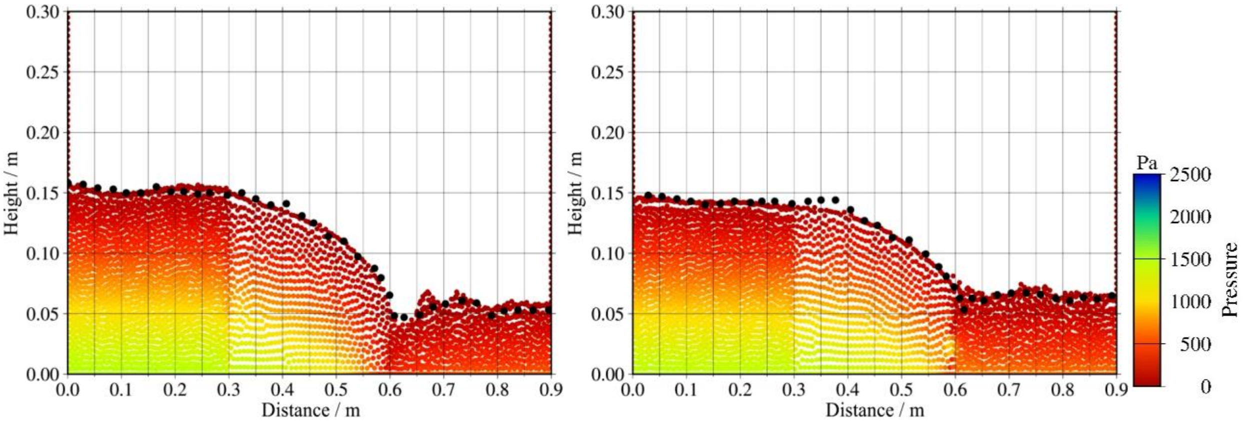
where 0 denotes the initial time step, $\theta$ is the combination ratio and $W_{ij}$ describes the kernel function. In Equation (7), the first term is the HS and the second one is the standard source term. In the following simulations, $\theta$ is optimized at 0.97.



(a) T=0.0 s (b) T=0.4 s



(c) T=0.8 s (d) T=1.2 s



(e) T=1.6 s (f) T=2.0 s

Figure 2 Comparison between ISPH simulation (coloured dots) and Liu et al. (1999) experimental data (black dots)

### Fluid-porous Interface

In the present model, porous media is described by dummy particles which have porosity information. These dummy particles are referred only when the permeability and density are calculated at each time step based on the SPH formulae. Density of the target particle inside the porous region must be updated considering solid skeleton of porous structure if it is not submerged. One of the typical modification method is apparent density (e.g. Akbari (2014); Ren et al. (2016)) whose concept is to vary density depended on porosity.

$$V_j = \frac{m_j}{\rho_j}; \qquad \rho_j = Nw_j \rho_w \tag{8}$$

where $\rho_w$ represents fluid density. In this study, porosity $Nw_j$ varies linearly with the density $\rho_p$ inside the porous region in the following equation:

$$Nw_j = 1 - (1 - Nw)\frac{\rho_{pj}}{\rho_w} \tag{9}$$

Hence, the apparent density also changes linearly and no transition zone is needed. Ikariet al. (2017)developed a similar method for Moving Particle Semi-implicit (MPS).

## MODEL VALIDATION

The presented model is validated by the comparison with the dam-breaking experiment conducted by Liu et al. (1999). The numerical set up is described in Figure 1. The water column is 0.28 m wide and 0.25 m high. The porous block (*Nw*=0.49, *Dc*=1.59 cm) is placed at the centre of the tank. Figure 2 shows the comparisons of the free surface with the experimental data. The volume of the particles inside the block apparently looks to be increased and the model is successful in reproducing the solid skeleton of the porous media. Although the simulated free surface does not move much towards the right wall compared with the experimental data until *T*=0.4 s, the results are overall in good agreement with the experiment.

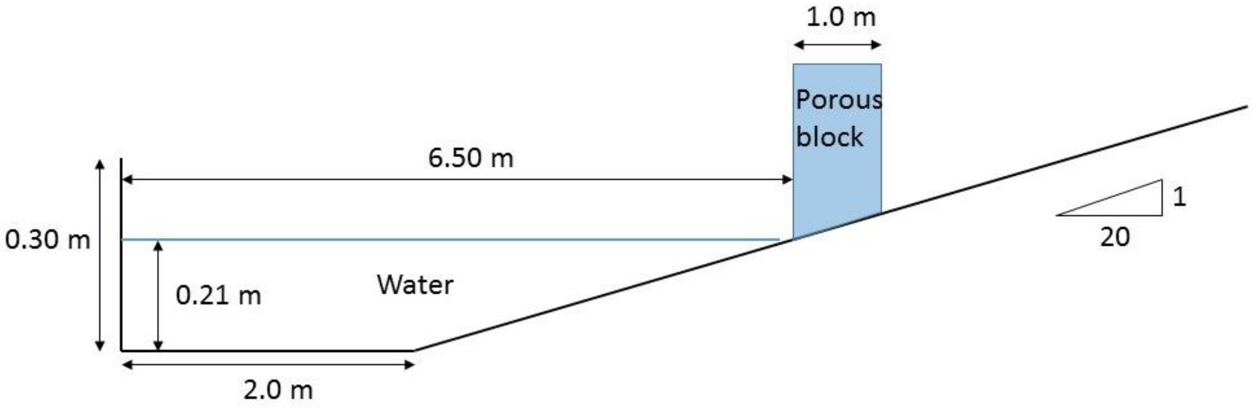
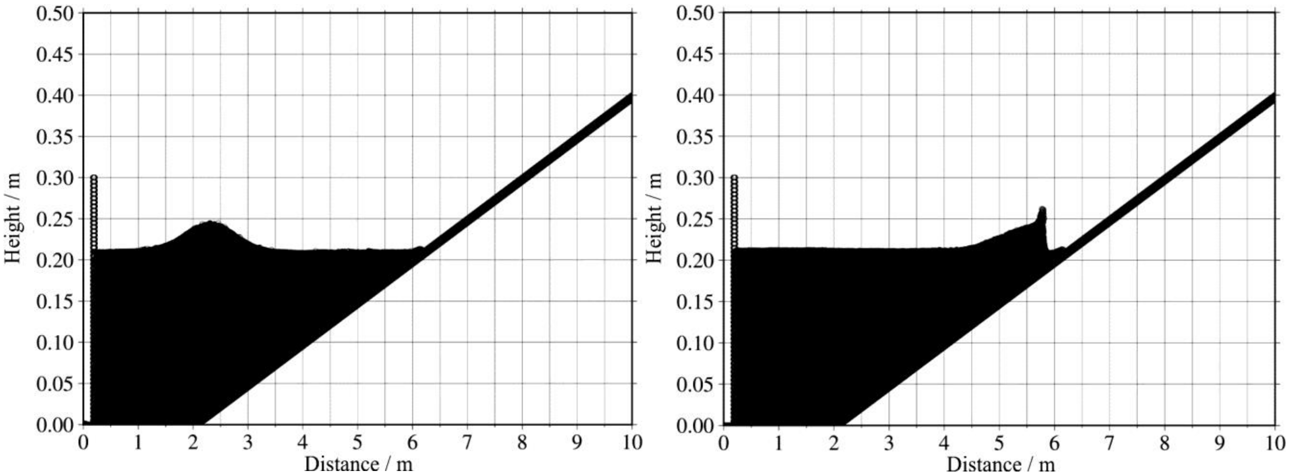


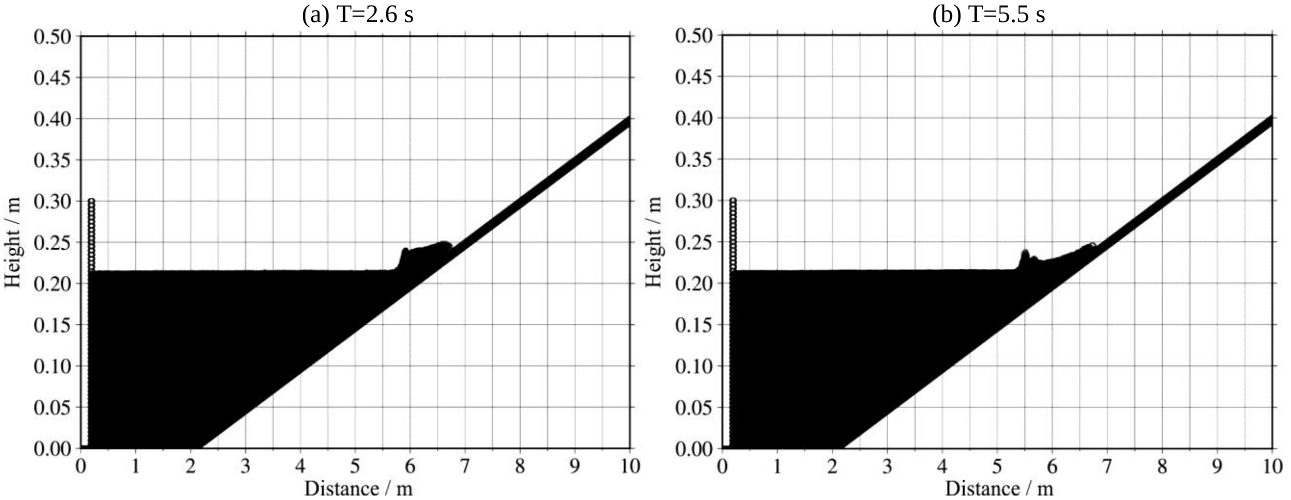
Figure 3 Schematic diagram of the simulation of wave run-up on slope with a porous block.

## MODEL APPLICATION

The present model is applied to solitary wave run-up on sloped beach. The computational domain is described in Figure 3. The slope scale is 1:20, and this is almost the same as 1:19.85 scale of the flume which Synolakis (1986) used for his run-up experiments. The porous block (*Nw*=0.49, *Dc*=1.59 cm) is placed on the beach at *X*=6.5~7.5 m assuming a coastal protection. The initial water depth is 0.21 m and the generated wave height is 0.03339 m so that the wave height ratio *H/h* becomes 0.159. The simulation results are presented in Figure 4. The solitary wave was generated by the paddle movement at the left wall and the wave broke while it propagated on the beach. The porous block weakened wave run-up and the run-up height ratio *R/h* is 0.17. Synolakis (1986) reported experimental data that *R/h* became 0.384 when *H/h* was 0.159. The mitigation effect of the porous block is evaluated as 56%.



(a) T=2.6 s (b) T=5.5 s



(c) T=7.5 s (d) T=8.0 s

Figure 4 Snapshots of the solitary wave run-up on sloped beach through a porous block.

## CONCLUSIONS

The ISPH model is developed for studying the water and porous material interactions. Porous particles are introduced in the porous region of the computational domain to describe the porosity information. They also represent the solid skeleton of porous media. The apparent density between the porous region and pure fluid region varies linearly. The model's predictions showed good agreement with the past experiments when applied to the wave run-up problem. It is expected that this model will be further applied to study various simulations involving the water wave interaction with porous materials, such as solitary wave propagation through soil and coastal forests.

## ACKNOWLEDGEMENTS

We are grateful for the UK-China Urban Flooding Research Impact Programme sponsored by the Royal Academy of Engineering (No. UUFRIP\100051).

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