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The Material Point Method for geotechnical engineering: A review for practical applications

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ABSTRACT

The material point method (MPM) has become increasing popular in geotechnical engineering in academia and in industry. There has been substantial progress in the development of MPM in the two years, including the interpolation techniques, the modelling of multi-phase materials and the development of contact conditions. However, there is still progress to be made for the method to become a daily engineering tool in practice. This includes the availability of the codes to industry, the documentation, benchmarking and reference case studies. This paper reviews the current state of development of MPM.

KEYWORDS: material point method; geotechnical engineering; engineering practice; computational geomechanics; numerical analysis.

INTRODUCTION

Since the early work of Sulsky et al. (1994; 1995), the material point method (MPM) has become increasing popular in geotechnical engineering, although largely in academia and a little in industry. This popularity is largely due to its proximity to the finite element method (FEM), which are both based continuum mechanics concepts, and because MPM can provide solutions where the conventional FEM faces some limitations. The proximity between MPM and FEM makes MPM more attractive for practitioners than other large deformation methods. Although there has been substantial progress in the development of MPM in the past years, it still requires some development in order to meet the requirements for its use as a daily tool by practitioners. This paper reviews some of the recent developments and gives some of the desired developments for engineering practice.

OVERVIEW OF RESEARCH PROGRESS

In the past few years, there has been substantial progress in MPM with advances both the numerical method itself and in the applications of the method for engineering practice.

Numerical method

The numerical method is the most important part of the code as it defines how the governing equations are solved in order to provide a solution to a given problem. In the past years, there has been a number of improvements in the formulations of the *standard* MPM (Sulsky et al., 1994; 1995) in order to overcome some numerical issues. One of these issues is related to material points (MPs) moving from one cell to another, which it is often referred to as *grid crossing error*. The continuous back and forth mapping of information between the MPs and the nodes has certain computational cost as well as causing some numerical issues (Brinkgreve et al., 2017). This results in unbalanced forces and stress oscillations. Many formulations of MPM mitigate this issue by using different interpolation techniques. These are for example the *generalised interpolation MP* (GIMP) (Bardenhagen and Kober, 2004), the *convected particle GIMP* (cpGIMP) (Sadeghirad et al., 2011; 2013), the *dual domain MP* (DDMP) (Zhang et al., 2011), the *B-spline based MP* (BSBMP) (Bing, 2017; Steffen et al., 2008; Tielen et al., 2017), the *conservative Taylor Least Square reconstruction MP* (CTLSR-MPM) (Wobbes et al., 2019, 2018), or the *moving least squares MP* (Hu et al., 2018) methods. There has also been progress in the definition of the time steps with *temporally adaptive MP* (Fang et al., 2018). Although these new formulations improve the accuracy of the MPM for specific applications, they also significantly increase the computational cost (Kularathna, 2018) making MPM computationally expensive. Computational capacity has always been a limitation for numerical methods alongside knowledge deficiency in computational mechanics (Hoek, 2015).

In the recent years, there has also been attempts to coupled different numerical methods together in order to gain the advantages from one and the either. For instance, Lian et al. (2012) and Lim et al. (2014) showed cases in which both MPM and FEM computations can be carried out within the same simulation, which decreases the computational demand by skipping the convective cycle for small strained areas. X. Zhang et al. (2017) give a detailed description of the coupling between MPM and FEM. Higo et al. (2010) coupled MPM with the finite difference method (FDM) and Raymond et al. (2018) with smoothed particle hydrodynamics (SPH). Liu et al. (2018) coupled both MPM and the discrete element method (DEM) to model granular flows and in which MPM is used to model the flow and DEM to capture the micro-mechanical behaviour of individual blocks.

Formulation for geomaterials

Different formulations of MPM are necessary to model geomaterials due to their porous nature and the influence of pore fluid pressures on their mechanical behaviour. Stomakhin et al. (2014) developed a formulation to model the transition between solid and liquid phases of a given material, and Yerro et al. (2015) a formulation for partially saturated soils in which the governing equations are solved for the solid, pore liquids and pore gases and illustrated in Yerro & Alonso (2019). Bandara et al. (2016) also modelled partially saturated landslides using the one-phase single-point formulation (Fig. 1) but they neglected the pressures, densities and relative accelerations of the fluid phases. More recently, Stomakhin et al. (2014) included the temperature as a variable to model melting processes. Pinyol et al. (2018) demonstrated the importance of temperature when modelling the run-out distance of landslide and illustrated in Alvarado et al. (2019). Yerro et al. (2018) demonstrated the use of a single-point formulation for assessing the run-out distance of the 2014 Oso Landslide. However, these formulations are based on the single-point approach in which all the information required for the computation is held within one set of material points. Bandara & Soga (2015) developed a double-point formulation in which two sets of material points are used to represent different phases and this approach considers the relative acceleration between the different phases. This formulation is especially important as it permits modelling not only pore water within the soil but also free water (e.g. reservoirs) and its interaction with soil. This permits modelling the failure mechanism of dam and levee over-topping. This formulation was further improved by Martinelli (2016). Fig. 1 depicts the different MPM formulation for geotechnical engineering.

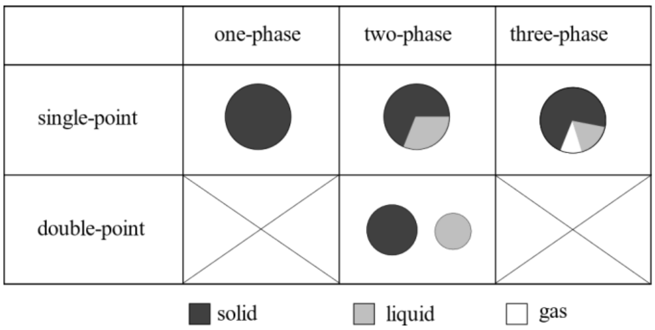

Figure 1 The different MPM formulation for geotechnical engineering (after Yerro & Rohe 2019)

The ability to model different phases with different sets of material point has a strong practical significant as it opens new engineering opportunities in geotechnical engineering. For instance, Hamad et al. (2017) and Coelho Zuada (2019) demonstrated the use of MPM for modelling the installation process of geomaterials taking into account the stiffness and permeability of the geocontainers as well as the water flows around the geocontainers. Liang et al. (2017) simulated the erosion process of a soil bed subjected to a water jet and Zhao & Liang (2019) the scouring process of a submarine pipeline. There has also been significant development in the field of deep borehole drilling. So far, the incompressible materials were modelled as near-to-incompressible, which causes a number of numerical issues. Kularathna & Soga (2017a; 2017b) revised the formulation in order to account for fully incompressible fluids in absence of any stress oscillation. F. Zhang et al. (2017) also revised the method to consider incompressible free surface flows.

Integration scheme

Many MPM codes are implemented with explicit time integration schemes and rely on the Courant-Friedrichs-Lewy (CFL) condition to determine the critical time step. Mieremet (2015) and Ceccato (2015) reviewed the CFL conditions for the double-point formulations (Fig. 1). An implicit formulation allows for larger time steps and thus permits modelling longer time periods as demonstrated by Fern et al. (2017) when modelling the failure of levees. However, this requires a more complex assembly and solution algorithm (Brinkgreve et al., 2017) leading to more computational cost. However, this additional cost can be compensated by larger time steps and less computation cycles. Since the early work of Cummins & Brackbill (2002), Guilkey & Weiss (2003) and Sulsky & Kaul (2004), new implicit schemes have been suggested, e.g. implicit GIMP (iGIMP) (Charlton et al., 2017).

Model layout

Many MPM codes are formulated for three-dimensional and two-dimensional plain strain configurations. Some years ago, Sulsky & Schreyer (1996) suggested a two-dimensional axisymmetric formulation, which was more recently improved by other by Hammerquist & Nairn (2018) and Galavi et al. (2019).

Anti-locking techniques

Locking effects are mismatches between the kinematic deformation possibilities and flow rules, which can lead to unrealistic overshoots of stresses and are of practical relevance. Mast et al. (2012) proposed using the Hu-Washizu principle to overcome locking issues. Bandara & Soga (2015) suggested using the B-bar technique, and Brinkgreve et al. (2017) a second-order calculation grid suggested to the smoothness of the displacement and velocity field. Coombs et al. (2018) suggested a method for both the *standard MP* and *GIMP* methods, and Yang et al. (2018) presented a flux-based smoothing algorithm based on controlled strain energy dissipation.

Contact conditions

Contact conditions control how forces are transmitted between two bodies in contact. They are the MPM equivalent of FE interface elements. These algorithms must first detect contacts, locate them and determine the contact forces. Bardenhagen et al. (2000) suggested the first contact algorithm, which was later improved by Huang et al. (2011) to model impact problems. Ma et al. (2014) developed a contact formulation to model fully and partially rough contacts by including a penalty function to reduce velocity and acceleration oscillations in the region of the contact surface. Al-Kafaji (2013), followed by Ceccato et al. (2017), included an adhesion term to simulate for CPT simulations. Jiang et al. (2017) proposed a contact formulation for non-volumetric (very thin) objects. Homel & Herbold (2017) developed a formulation for brittle materials in which cracks propagate allowing for frictional displacements. Yang et al. (2018) revised the contact formulation for soil-structure interactions and Nairn et al. (2018) generalised the contact formulation to consider more advanced contact models, including velocity-dependent models and heat generation.

Contact conditions are necessary in geotechnical engineering in order to properly model soil-structure interactions. Examples of applications are the cone penetration tests (e.g. Ceccato et al. 2016; Ceccato & Simonini 2019; Konkol & Ba 2017), pile installations (e.g. Phuong et al., 2016; Rohe and Nguyen, 2019), or sliding and flowing landslide masses (e.g.. Yerro et al., 2018).

Boundary conditions

As much as the MPM formulations are necessary to obtain accurate solutions, many numerical features are still required in order to idealise correctly a given engineering problem. These features are typically conditions imposed to parts of the model in order to model engineering processes. Although these features seem at first trivial developments, they are necessary for practical applications. Cortis et al. (2018) extended the implicit essential boundary condition for inclined boundaries allowing a reduction in the model size and, hence, decreasing the computational cost. Zhao & Liang (2019) created new in- and outflow boundary conditions to model steady-flow systems of fluid. This allows creating and removing material points from the system and, subsequently, to model overtopping problems of dams and levees. It also allows simulating excavation processes. Two examples of excavation-induced instabilities are given by Pinyol & Di Carluccio (2019).

Modelling geomaterials and structural elements

There has also been significant progress in modelling complex behaviours of geomaterials. For instance, Gaume et al. (2018) demonstrated the ability of MPM to model the on-set and post-flow behaviour of snow avalanches by adapting a critical-state constitutive model for snow. Fern & Soga (2016) showed that the choice of the constitutive model influenced the energy dissipating mechanism and, hence, the run-out distance of granular flows. Coelho Zuada & Nuttall (2019) showed that this choice also influences the factor of safety for stability analyses of levees. The determination of the constitutive model is case specific and there is no guidance on which model to use. Fern (2016) pointed out that many MPM simulations presented in the literature rely on simple failure criteria as constitutive models, also referred to as *first generation of models* (Potts et al., 2002), for which the implementation in MPM is discussed in X. Zhang et al. (2017). Pinyol et al. (2018) and Alvarado et al. (2019) improved the ability of MPM to localise strain and to consider the generated heat.

Numerical predictions are hampered by the difficulty of making accurate predictions due to inherent variability and uncertainty of the ground conditions. For this reason, Wang et al. (2016) and Remmerswaal et al. (2019) included the random field theory in MPM, which they called the *random material point method* (RPMP). This method considers the spatial variation of parameters based on statistical analyses. They demonstrated the use of RPMP for two-dimensional plain strain and three-dimensional cases, respectively. They subsequently simulated rainfall-induced slope failures (Wang et al., 2018).

There has also been progress in modelling structural elements. Guo et al. (2018) successfully modelled the buckling of cylindrical shells. Although MPM can model the behaviour of structural elements, there has been very few examples of soil-structure interaction in geotechnical engineering. In many cases, the structural elements are modelled as rigid bodies (i.e. Mast et al., 2014) or as continuum material (i.e. Rohe and Nguyen, 2019). Yang et al. (2017) modelled the interaction between tsunamis and retaining structures.

MPM FOR PRACTICAL APPLICATIONS

Although there has been substantial progress in the development of the MPM for geotechnical engineering, there is still a gap between the technical capabilities of the MPM codes in academia and their use in day-to-day practice. Brinkgreve et al. (2017) reviewed the use of MPM for practitioners and they identified a series of numerical difficulties which must be overcome. These are mostly numerical inaccuracies of the method and, more specifically, instabilities related to the integration schemes, contact formulations and the computational cost. As much as the formulation of the numerical method is paramount for accurate predictions, there are many other features necessary for a research code to become a successful tool for practitioners such as (1) documentations and guidance, (2) validation, benchmarking and reference cases, and (3) ease in use. The accuracy of a prediction also depends heavily on the knowledge of the site and ground conditions used to calibrate the numerical parameters (Fern et al., 2019b).

Potts (2003) raised the question: “Is numerical analysis a virtual dream or a practical reality?”. This question was motivated by a lack of popularity of numerical predictions in practice at that time, which was caused by a lack of documentation, expertise and guidance of FEM. A series of guideline documents have since been published (i.e. Lees 2016; Fern et al., 2019b; Potts et al. 2002; Potts & Zdravković 1999; 2001) discussing the process of carrying numerical predictions. Whilst some of the content is applicable to MPM, additional information is still required. X. Zhang et al. (2017) published the first book on MPM with an emphasis on the science of the method. Fern et al. (2019) published the second book on MPM with an emphasis on its practical use in geotechnical engineering. However, these documents only present the current state of MPM and its application, and there is still progress to be made and questions to be answered. For instance, Fern et al. (2019) showed that the choice of the constitutive model can strongly influence the results of slope stability analyses both in terms of run-out distance and factors of safety but there is still no general rule on which constitutive model to use. There is also a necessity to better model soil-structure interactions by introducing structural elements (i.e. beams, plates, anchors, struts, wells, etc.), to generalise the contact formulations, and to further developed the simulation of groundwater seepage and free water flows. It is also important to facilitate the use of MPM by providing users with graphical interfaces, which reduces the time required to setup a model and reduces the engineering cost. Computer capacity still remains a challenge for the use of MPM in geotechnical engineering practice.

Numerical methods are used in industry to predict the behaviour of problems with complex geometries and ground conditions in order to explore possible outcomes. Fig. 2 shows a schematic description of a levee before and after failure due to rising sea levels. The problem involves both groundwater and free surface water and, hence, a double-point formulation is required. The ground is composed of a wide range of materials with different states. As a three-phase double- and triple-point formulations do not exist, the contribution of the partial saturation on the stability of the system is neglected. A weak stratum of soil exists beneath the marsh and favours the sliding process. It has to be decided if this layer should be modelled as a thin continuum solid or as a contact condition. The levee is equipped with cut-off wall within the dam and an I-wall on the crest. These should be modelled as thin structural elements and a contact conditions must be applied in order to model to correct relative movement between the structure and the soil. It is unclear how the road should be modelled as its implication in the failure process is not clear. In the eventuality that it did play a role, the road should be modelled as a load applied to a slab resting on piles, and these structural elements should also have contact conditions in order to allow the soil to move around them. The post-failure configuration shows that strains localised in a thin layer forming a shear band. Thus, a fine mesh is required but it increases the computational cost of the simulation. Although this practical case appears at first to be simple, it turns out to be complicated due to interaction between the water and the soil, the numerous soil units with different mechanical behaviours, the structural elements and their contact conditions and the localisation of strains. Moreover, the failure surface of this case (Fig. 2) has three different curvatures making it quasi-impossible to model with conventional limit equilibrium software without making simplification of the failure mechanism or with FEM due to the large deformation of the problem.

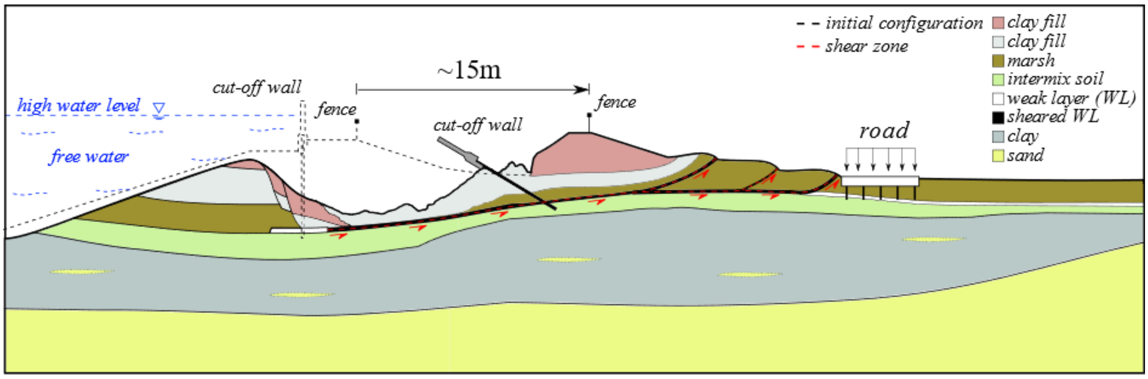

Figure 2 Schematic representation of a failed levee (modified from Seed et al. 2008)

CONCLUSION

A brief review of the scientific developments of the material point method (MPM) is presented with an emphasis on its practical use. MPM is an ideal tool for practitioners for modelling both large deformation problems, though it can also model small deformation problems, as it is based on the same mechanics as the finite element method (FEM), which is extensively used by practitioners.

It appears that substantial progress has been made in the stability of the method by developing different interpolation schemes as well as on its ability to model transitions from solid to liquid and *vice versa*. There has also been progress in the formulation of contact conditions between objects, which is necessary when modelling soil-structure interactions. However, only few studies investigated the interaction between structures, solids and fluids and they often only consider one object modelled as an elastic continuum or a rigid body. There is also a necessity to use multiple materials, structures and contact conditions in complex phased analyses for which an example was given. Documentation, benchmarking and guidelines can also be improved in order for MPM to be used in everyday geotechnical engineering.

MPM is a powerful engineering tool, which open up new engineering opportunities. However, it is not a replacement for blind judgement. Despite significant progress in MPM in the past years, some major issues still remain. These include data and knowledge deficiencies and limited computer capacity.

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