

## **Direct Numerical Simulations on local scour around bridge pier with advanced Immersed Boundary Method**

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### **ABSTRACT**

Local scour is one of the major causes of the bridge failures and the numerical simulation plays a leading role in studying the development of scour holes with its advantages of flexibilities and precisions. In this paper, direct numerical simulations with the Eulerian-Eulerian approach for solving the problems of sediment transport are employed to study the local scour near the bridge. An advanced immersed boundary method is developed to capture the complex geometry of scour hole dynamically. In order to ensure the stabilities and accuracies, an adjusting method based on the angle of repose is used to modify the bed levels which are impractical due to the fluctuations of flow velocities. The results of numerical simulations on suspended sediment concentration and the geometry of the scour hole show great agreements with the theories and the experiments.

**KEY WORDS:** Direct numerical simulations; Local scour; Immersed boundary methods; Adjusting methods; Sediment transport;

### **INTRODUCTION**

Scouring is one of the major causes of a catastrophic failure in bridge piers around the world and has also happened in China as in the Yangtze River, and in the UK as in North England about a decade ago. This is despite having a few standards on scouring and its danger to piers. Scouring must be accounted when designing piers in order to stand various conditions of water flows (Breusers et al., 1977).

Scouring is a result of sediment transport locally increasing near the pier. Various empirical equations for calculating sediment transport have been proposed following a large quantity of experimental studies (Kumar et al., 2012) and some of these have been widely used in numerical simulations. All of these established relationship between sediment transport and characteristics of flow as shear stresses or velocities near the bed. In numerical studies two approaches can be used to simulate the sediment transport; Eulerian-Eulerian and Eulerian-Lagrangian, where the first word in the hyphenated term refers to the turbulent flow simulation and the other to the particle flow simulation. Although the Eulerian - Lagrangian approach can yield high accuracy for sediment transport (Ji et al, 2014) it is highly computationally expensive. On the other hand, the Eulerian-Eulerian approach requires less computational resources when modelling the sediment transport through using a-priori relations (Singh et al 2017). It is worthwhile to put forward the Eulerian-Eulerian approach for engineering design because the approach can achieve a great balance between the computational times and the accuracies. At present, many research carry out their numerical experiments on local scour based on Eulerian-Eulerian approach (Bakhtyar et al., 2009), but few of them use direct numerical simulations which are more accurate than other turbulent models such as RANS models. What's more, the problems that how to capture the complex scour's geometry accurately during the direct numerical simulations require further study.

In this study, known sediment transport laws and mass balance in order to predict the time-development of the scour's geometry are used in the direct numerical simulations (DNS) based on the code CgLES. The complex geometry of the scour will be captured using an advanced immersed boundary technique, which consists of a method of movable immersed boundary points and a method of adjusting the bed level according to the angle of repose of sediment particles. The numerical methods are introduced in the next section. Then the verifications and the results of the numerical model are followed. And the last section is the conclusion of the numerical simulations.

## NUMERICAL MODELS

### Hydrodynamic Models

Navier-Stokes equations and continuity equation govern the three-dimensional unsteady incompressible flow in the direct numerical simulations:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \cdot \nabla^2 \mathbf{u} + \mathbf{f}, \quad (2)$$

where  $\mathbf{u}$  is the velocity vector;  $\nabla$  is the Laplace operator;  $t$  is the time,  $p$  is the pressure;  $\nu$  is the kinematic viscosity, which is equal to  $10^{-6}$  m<sup>2</sup>/s at the normal temperature of 20 °C, and  $\mathbf{f}$  is the body force, which is equal to the gravitational acceleration in the vertical direction. No turbulence model is used for adopting DNS.

The projection method is used to solve the Navier–Stokes equations in the code CgLES, taking the divergence of the momentum equation and solving the pressure Poisson equations (Thomas et al., 1994). The explicit Adams–Bashforth scheme is employed to obtain second-order accuracy in time to discretize the momentum equations. Second-order backward difference in space is employed to discretize the advection terms, diffusion terms, body force terms, and pressure gradient terms.

### Sediment Transport Models

In general, sediment transport can be divided into bed load transport and suspended load transport. The calculation of bedload transport is based on Van’s studies (Rijn, 1984b):

$$q_b = 0.053 \frac{T^{2.1}}{D_*^{0.3}} [(s-1)g]^{0.5} D_{50}^{1.5}, \quad (3)$$

The formula can be used to calculate the equilibrium bedload transport  $q_b$ , where  $T$  is the transport stage parameter;  $D_*$  is the dimensionless particle parameter and the  $D_{50}$  is the median particle diameter;  $s$  is the relative particle density;

The suspended sediment is governed by the advection-diffusion equations:

$$\frac{\partial C}{\partial t} + \nabla \cdot [(\mathbf{u} - \mathbf{w}_s)C] = \frac{\partial}{\partial x} \left( k \frac{\partial C}{\partial x} \right) + S, \quad (4)$$

where  $C$  is the volume concentration of a solute;  $w_s$  is the settling velocity of the sediment (Rijn, 1984d);  $k$  is the molecular diffusivity coefficient, and  $S$  is the source term, which exists only at the bottom boundary (named  $S_b$  below). The source term is set to describe the erosion and the deposition of the bed level based on the pick-up functions in accordance with Van’s studies (Rijn, 1984a):

$$\phi_b = 0.00033 [(s-1)gD]^{0.5} D_*^{0.3} T^{1.5}, \quad (5)$$

$$S_b = D - E = \beta_1 \phi_b - \beta_2 \phi_b, \quad (6)$$

where  $\phi_b$  is the pick-up rate of suspended sediment from the bed and the amount of this depends on the critical Shields number and the shear stress at the bottom of bed;  $\beta_1$ ,  $\beta_2$  are the coefficients of erosion and deposition respectively.

### Fluid-structure Interaction Methods

The Immersed Boundary Method (IBM) is well-understood method and has been employed in the code CgLES and many numerical simulations based on the IBM like flow around piers have been carried out (Ji et al., 2012). The traditional IBM can successfully describe the interface between the fluids and structures by adding some static IBM points in the computational domain. In this study, the advanced IBM is developed and the IBM points are divided into two types; one is static to describe the pier and the other is movable to characterize complex scour’s geometry near the pier. Avoiding frequent re-meshing, the flow equation can instead be solved on a fixed grid during the development of scouring by using the advanced IBM. The change of bed level is based on the mass balance law (Xiong et al., 2016):

$$(1 - \alpha) \frac{\partial z_b}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} + S_b = 0, \quad (7)$$

where  $\alpha$  is the porosity of the bed;  $z_b$  is the bed level characterized by the movable IBM points;  $x$ ,  $y$  are the horizontal directions and  $q_{bx}$ ,  $q_{by}$  are the components of  $q_b$  in the two direction.

### The Adjusting Methods Based On the Angle of Repose

In the DNS, the velocities and shear stresses at the bottom of bed usually fluctuate strongly. Furthermore, it will lead to impractical changes of bed level during the computation such as an erratic scour hole, which not only is of no significance but also influences the stabilities of the computation. As a result, a new adjusting method based on the angle of repose is proposed to solve the problems. The fundamental theory of the method is based on the fact that if the slope of bed is larger than the critical slope controlled by the angle of repose, the slope will be unstable and the particles at the top keep sliding off until the slope of bed. The adjust method mainly contains the following steps: 1) For each cell, calculate the slope of bed according to bed levels its adjacent cell and mark the unstable slopes; 2) Divide the each cell into different types on the basis of the number of the unstable slopes; 3) Sequence the unstable slopes in terms of angle of repose and adjust them one by one; 4) Repeat the Steps above. In this study, the sizes of the grids are uniform, so the mass balance law can be simplified as follows when adjusting the bed level:

$$z_{b_i} + \sum z_{b_j} = 0, \tag{8}$$

where  $z_{b_i}$  is the bed level of the  $i$  cell;  $z_{b_j}$  is the bed level of all the adjacent cells. On the other hand, the method takes the fact into the consideration that the particles at the top of the unstable slope will slide off simultaneously. Consequently, the changes of bed levels of all the adjacent cells with unstable slopes in each iteration yield at:

$$\frac{z'_{b_j}}{(\tan \theta_0)^{-1.5} - (\tan \theta_j)^{-1.5}} = K, \tag{9}$$

where  $z'_{b_j}$  is the change of bed level of  $j$  cell with unstable slopes in each iteration;  $\theta_0$  is the angle of repose and  $\theta_j$  is the initial unstable slope angle of  $j$  cell.  $K$  is a constant parameter independent of adjacent cells.

### MODEL VALIDATIONS AND RESULTS

#### Profiles of Suspended Sediment Concentrations

The numerical model has been validated by many researchers in terms of the flow solvers and the IBM in the code Cgles (Zhu et al., 2017). As a result, this subsection is only focused on the validations of models of sediment transport. In the cases of validations, the computational domain and the grid size are set as shown in Table.1, where  $x$  is the flow direction;  $y$  is the transverse direction;  $z$  is the vertical direction;  $N$  is the number of grids;  $\Delta x^+$  is the dimensionless grid size based on the shear velocities; The boundary conditions of inlet and outlet, together with the side boundaries, are set as periodic conditions. The bottom boundary is no-slip boundary and the free surface boundary is seen as rigid lid condition. The flow Reynolds number is 225000 and the particle Reynolds number is 588 (based on  $D_{50} = 0.385$  mm).

Table 1 Parameters of the computational domain

$L_x$	$L_y$	$L_z$	$N_x \times N_y \times N_z$	$\Delta x^+$	$\Delta y^+$	$\Delta z^+$
$6H$	$2H$	$H$	$512 \times 256 \times 256$	4.2	2.3	1.4

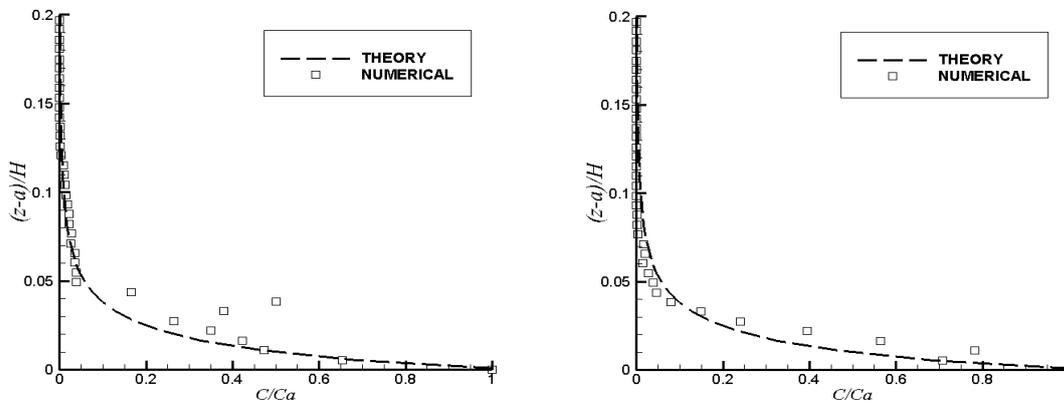


Figure 1 The profiles of suspended sediment concentration among the two vertical investigate lines

The results of numerical simulation are shown in Fig.1. As shown in Fig.1, it is the plot of relative suspended sediment concentration varying with the water depth at the time of sufficiently developed suspension. Here,  $z$  is the vertical coordinate;  $a$  is the reference height, which is usually defined as  $\max\{0.01h, 2D_{50}\}$ , where  $H$  is the

total water depth;  $C$  is the concentration of suspended sediment and  $Ca$  is the concentration of suspended load at the reference height. The theoretical profile determined by Rouse number mainly takes settling velocity and shear velocity into consideration (Rijn, 1984c). Fig.1 shows good consistency between the numerical results and theoretical results despite have some little difference, which is probably caused by the turbulence of the flow.

**Geometry of Scour holes**

In order to study how the model works on simulating local scour near the bridge site, a numerical tank is set the same size as the study of Melville’s laboratory experiments, with 2.0m long, 0.46 m wide, 0.15 m deep and a cylindrical pier whose diameter is 5.08cm set at the central line of the tank at  $x = 0.7$  m. The median diameter of sediment is 0.385mm and the mean flow velocity is 0.25m/s as the Melville’s study set (Melville, 1975).

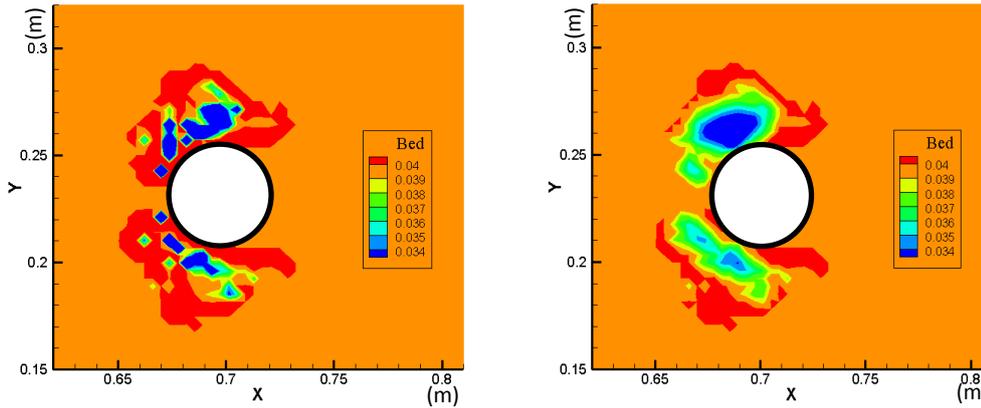
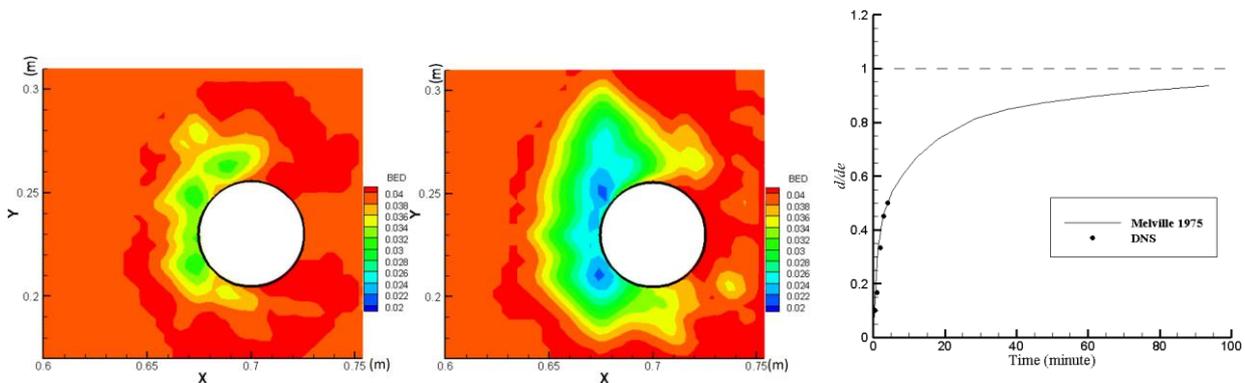


Figure.2 Contours of bed level around the pier after 30-second scouring  
a) Left: original; b) Right: applied adjust method

The Fig.2 shows the impacts of the adjusting method on the geometry of local scour. The left panel is the results of DNS in which the adjusting methods are not applied. As seen in the Fig.2a, the differences in bed levels between the adjacent cells are very large. These errors of bed level will not be diminished with the development of computation and it will finally lead to the failure of stable computations. By means of employing the adjusting method, the results of scour become more reasonable and it can avoid the instabilities during the simulation. The Fig.3a and Fig.3b show the geometry of bed after some-time scouring. It can be found that the scour begins at the front side of the pier and then the scour hole extends downstream. With the time going on, the scour hole is bigger and deeper. The maximum depths of scour hole varying with time in the DNS are extracted and a comparison is made with the results of Melville’s laboratory experiments. As shown in Fig 3.c, although the scour hole in the DNS is still developing, the relative maximum depth of scour hole (nondimensionalized by the equilibrium depth 0.06m in the Melville’s study) agrees with the laboratory results well.



a) Left: 1-minute scouring; b) Middle: 2-minute scouring; c) Right: the maximum depth of scouring hole varying with time  
Figure.3 The geometry of scouring hole in the DNS

**CONCLUSION**

In this study, in order to capture the complex geometry of the local scour hole near the pier, the advanced immersed

boundary method and the adjusting method based on the angle of repose are employed in the direct numerical simulations. It can be found from the results that the adjusting method plays an important role in direct numerical simulations because it can avoid the instabilities of computation caused by the sharp slope of bed level. Despite lack of computational time, with the validations of suspended sediment transport and the comparisons of local scouring hole with Melville's experiments, it can be concluded that the established Eulerian-Eulerian approach based on the advanced immersed boundary method in this study is accurate enough for further study on the local scours.

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