



## **Nested manifold MPM simulations for modeling fluid-driven fracture propagation**

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### **ABSTRACT**

A new method for modeling fluid-driven fracture propagation using the Material Point Method (MPM) is introduced. The fractured medium is modeled using standard MPM methods with explicit fractures. The fluid inside the fracture is modeled with a nested lower dimensional MPM model using the manifold provided by the explicit fracture. The full simulation and the nested simulation are coupled to model the behavior of the fluid-filled fracture.

**KEY WORDS:**MPM, manifold, hydraulic fracturing

### **INTRODUCTION**

The Material Point Method (MPM) has emerged as a valuable tool for modeling the interaction of natural and hydraulic fractures in unconventional reservoirs. Modeling and understanding the hydraulic fracturing process are important, both to optimize the production and economics of unconventional reservoirs, and to ensure a safe process respecting the environment. MPM is particularly useful for modeling these reservoirs because it can handle multiple distinct materials (Nairn 2007), accurately model discontinuities such as material interfaces (Nairn, 2007, Aimene et al., 2018) and fractures including dynamic fracture propagation (Nairn, 2003; Guo, Nairn, 2004; Guo, Nairn, 2006), allows for arbitrarily complex material models such as anisotropic damage mechanics models (Nairn, Hammerquist and Aimene, 2017), can include fully coupled poroelasticity (Khodabakhshnejad et al, 2017) as well as viscoelasticity (Peterson et al. 2018) and can natively handle solid-fluid interactions (Hammerquist, Nairn 2018).

Fluid driven fracture propagation is an important and complex multiscale physics problem. In a previous attempt to model these physics with MPM, the fluid inside the fracture was handled directly as a fully modeled material that interacted with the fracture surfaces with contact (Raymond et al, 2015). The multi-scale nature of reservoir modeling makes this approach useful to highlight the observation at the fracture scale. The resolution needed to accurately resolve the fluid flow requires a specific treatment to accommodate large reservoirs simulations. Here we introduce another technique to model this problem, a nested lower dimension MPM simulation along the fracture.

### **METHODS**

A method named CRAMP was developed (Nairn 2003) for explicitly modeling fractures in MPM. In a 2D CRAMP simulation, a fracture is represented by a series of massless fracture particles connected with line segments. An example of a CRAMP fracture is drawn in Figure 1. This assembly of particles and segments translates with the surrounding material and delineates the fracture discontinuity. Nodes near the fracture have multiple velocity fields on the grid, which allows for correct treatment of this discontinuity including handling contact of the fracture surfaces. The separate velocities at the nodes also allows for accurate tracking of the fracture surfaces which are needed for calculating certain properties including  $J$ -integral and stress intensity factors (Guo and Nairn 2006). Propagation is achieved by inserting a new fracture particle in front of the fracture tip. This formulation of fractures also allows traction laws, cohesive laws, or pressure be added to the

fracturesurfaces: the forces are calculated on the fracture particles and then interpolated to the grid. For 3D CRAMP simulations, the fracture surfaces are represented by fracture particles connected with triangle elements (Guo and Nairn 2017).

To model the effect of a hydraulically driven fracture in MPM, pressure can be applied to the fracture surfaces. But this pressure needs to be calculated from the physics of the problem. To accomplish that, we added a nested MPM simulation to model the fluid flow inside the fracture. The assembly of fracture particles and segments form a manifold on which to run this nested simulation.

For example, in a 2D CRAMP simulation, the fracture is a line or curve forming a 1D manifold embedded in the 2D simulation space. Then a 1D MPM is run on this manifold, using the fracture points as grid nodes for the nested simulation. The normal 2D MPM simulation models the behavior and response of the fractured medium. The flow inside the fracture is modeled in 1D with a separate nested MPM simulation. In a 3D MPM simulation, the fracture surface would form a 2D manifold. All the work presented here is with a 2D solid/1D fluid.

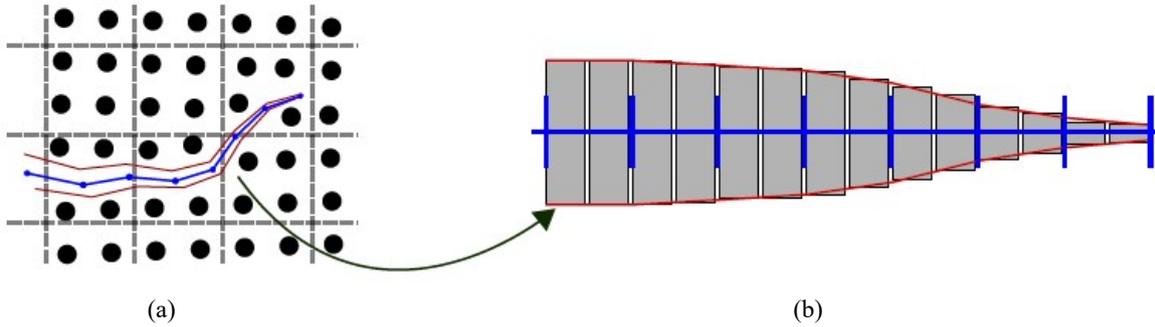


Figure 1 (a) An explicit fracture in a 2D MPM simulation. The fracture plane is shown in blue with the fracture particles shown as the blue dots and the fracture surfaces as red lines. (b) The corresponding 1D simulation on the manifold provided by the fracture. The grid corresponding to the grid nodes is shown in blue, the width is shown in red and the 1D particles as the grey rectangles

Since the embedded manifold might be curved and/or moving and stretching with the background material there will be additional terms in the momentum equation to solve including fictitious forces, but for our purposes, these are ignored. The velocity and stress are averaged in the dimensions not aligned with the manifold, leaving just pressure and velocity along the fracture plane. The nested simulation is close to a standard 1D MPM simulation with the additions of fracture width and viscous forces on the flow. In these 1D simulations, the fluid particles have both a length and a width. The width is determined by the opening of the outer fracture at that point. An example of this is shown in Figure 1. The deformation gradient tensor has 2 components,  $F_{11}$  and  $F_{22}$ . The first component  $F_{11}$  is updated through time in the normal way:

$$F_{11}^{n+1} = e^{\Delta \varepsilon \Delta t} F_{11}^n \quad (1)$$

But  $F_{22}$  is based on the width of the fracture at the time and position of the particle:

$$F_{22}^{n+1} = \frac{w^{n+1}}{w^0} \quad (2)$$

where  $w^0$  is the initial particle width, defined when the particle enters the fracture simulation. However, the fluid is not allowed to apply tension on the fracture surface, so if the surface pulls away,  $F_{22}$  is limited to keep the pressure from going negative. The fracture width partially determines the shape of the fluid particle and so the particles can become highly deformed. CPDI shape functions (Sadeghirad, Brannon, Burghardt; 2011) are used because they account for large particle deformation and can easily handle arbitrarily-shaped background grids. The viscous forces  $f_p$  on the fluid particle are calculated based on the assumption of a Newtonian fluid in laminar flow:

$$f_p = -12\mu V_p \frac{v_p}{w^2} \quad (3)$$

where  $\mu$  is the fluid viscosity,  $V_p$  is the particle's volume,  $v_p$  is particle velocity and  $w$  is the local fracture width

determined from the outer simulation. This force is then interpolated to the grid using the shape functions. This formulation is not restricted to Newtonian fluid laminar flow. It would be straightforward to add different viscous models, such as velocity dependent viscosity, or averaged turbulence models. A version of the Tait equation of state (Li, 1967) for water was used to calculate the pressure:

$$p = 0.0894K[\exp\left(\frac{1}{0.0894}(1 - J)\right)] \quad (4)$$

where  $J = F_{11}F_{22}$  is the Jacobian determinant and  $K$  is the fluid bulk modulus. The pressure in the nested simulation is applied to the fracture surfaces of the full dimensional simulation. The outer MPM simulation of the fractured medium and the nested manifold MPM simulation are dynamically stepped through time with an explicit integrator. As the fracture propagates, additional grid nodes are added to the nested simulation. Also, as the fluid flows along the fracture, additional fluid particles are added as needed. The mouth of the fracture can be set as either a pressure or a velocity boundary condition.

A simple example of a nested simulation is shown below in Figure 2. In this example, the fluid is forcing open a fracture in a soft material.

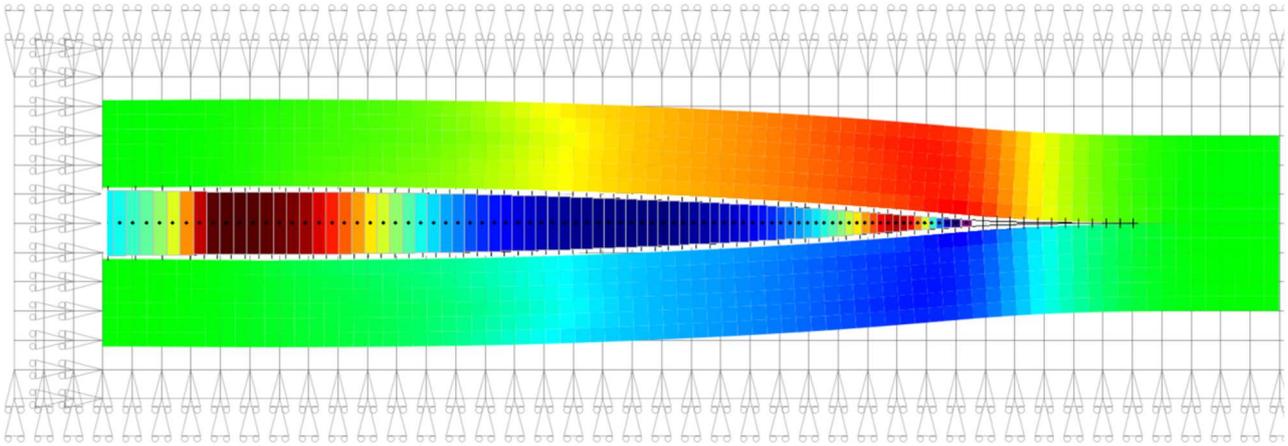


Figure 2 An example of a nested MPM simulation. The full outer simulation is a soft elastic material contained by grid boundary conditions and the colors show normalized vertical velocity. The color in the 1D particles show normalized pressure inside the fracture

## RESULTS AND DISCUSSION

To test the effectiveness of the proposed strategy, it is compared with an analytical model for a plane strain hydraulic fracturing proposed by (Geertsma J, de Klerk F., 1969), termed the KGD model. This model assumes an incompressible fluid at a constant flowrate driving a fracture in infinite elastic medium under a confining stress in plane strain conditions. Under these assumptions, the maximum fracture opening is given as function of time:

$$O_{\max} = 1.32 \left( \frac{8(1-\nu)\mu Q^3}{G} \right)^{1/6} t^{1/3} \quad (5)$$

where  $Q$  is flowrate,  $t$  is time,  $G$  is shear modulus,  $\nu$  is Poisson's ratio and  $\mu$  is the fluid viscosity. A nested MPM simulation was set up to approximate those conditions. An elastic region with a symmetry boundary condition in the middle and a confining stress of 1 MPA stress and a small initial crack was used. The material properties and dimensions are shown in Figure 3.

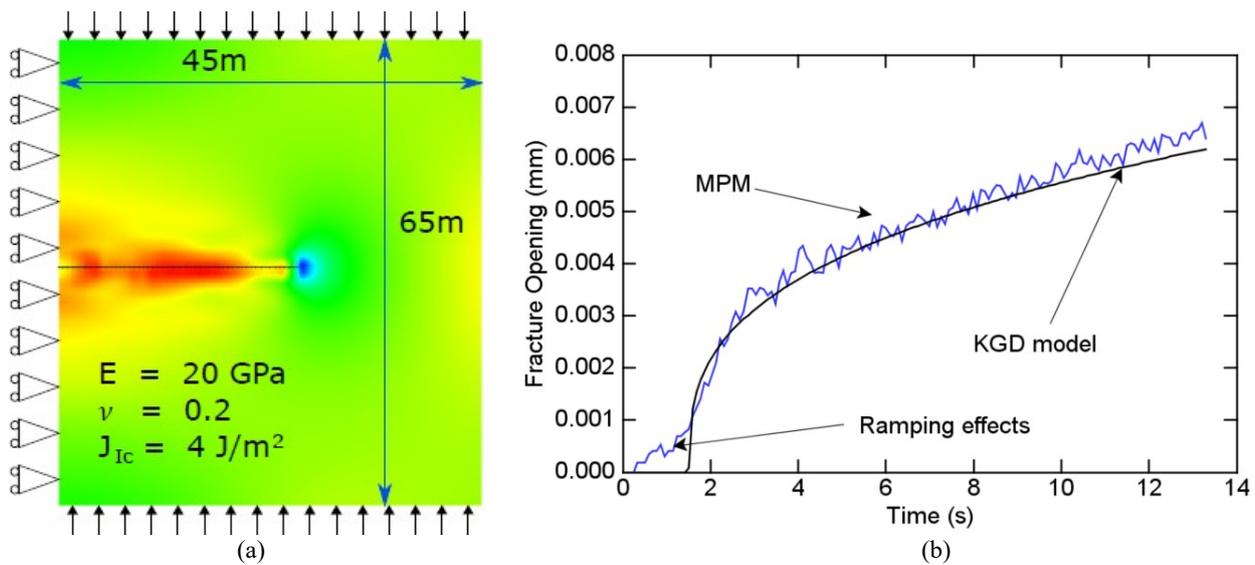


Figure 3 (a) The simulation parameters for the first test example. The color shows a normalized pressure at the end of the simulation. (b) Comparison of the crack mouth opening as a function of time compared with the analytical model. The analytical model is shifted in time to account for the ramping region in the MPM simulation

The MPM cell size used for the simulation was 1m. The fluid in the nested simulation had a viscosity of  $\mu = 0.0001 \text{ Pa} \cdot \text{s}$ , density  $\rho = 1000 \text{ kg/m}^3$  and bulk modulus  $K = 2 \text{ GPa}$ . Since this was a dynamic simulation, the flow rate at the inlet was ramped up to a constant value  $Q = 0.00001 \text{ m}^2/\text{s}$  per unit thickness at the beginning of the simulation to avoid infinite pressure at the beginning. Also, a small virtual preopening of the 1D simulation was added to the mouth of the fracture to allow for the fluid to start flowing into the gap. The fracture was allowed to propagate based on a  $J$ -integral criterion. A silent boundary condition was applied to the free edge and damping was used to dissipate the kinetic energy during the simulation. The fracture opening is compared with the analytical solution in Figure 3. The simulation results agree well with the analytical model, except for the beginning of the simulation due to the ramping of flowrate in the MPM simulation. As the fracture propagated deeper into the block, the assumption of “infinite medium” became less valid and the fracture opening value started drifting above that of the analytical model.

The fluid injection parameters are known to drive the fracture propagation path. This effect was tested on a non symmetric layered elastic medium. The medium consisted of a layered stiff layer bounded to a soft layer. The dimensions and material properties of this example are given below in Figure 4. The left edge was set to be a symmetry condition and the traction boundaries were used to apply a 2 MPa of compressive stress and the MPM cell size was 1m. Both layers had a Poisson’s ratio of  $\nu = 0.2$ , and a critical fracture energy of  $J_{IC} = 5 \text{ J/m}^2$ . There was small initial fracture in the middle at the left edge. The direction of the fracture propagation was controlled by the maximum hoop stress. The inlet condition for the nested simulation of fluid was a pressure boundary condition that was ramped with time, at a rate of 0.125 MPa/s. The fluid in the nested 1D simulation had a density  $\rho = 1000 \text{ kg/m}^3$  and bulk modulus  $K = 2 \text{ GPa}$ . Multiple simulations were run with different fluid viscosities. The fracture paths varied depending on the viscosity of the fluid and which layer the fracture was initiated from. Simulations with viscosity of 0.01Pa.s and higher, didn’t have any propagation for the same pressure. The different fracture paths are overlaid onto the same visualization below in Figure 4 for comparison. In all cases, the propagation started straight. Then as expected, when the fracture is initiated in a stiff material, the propagation path deviates to reach and cross to the soft material following the least resistant path. When the fracture is initiated in the soft material, the propagation path stays in the soft material. In both cases, higher deviations from the initial fracture path was seen with higher fluid viscosity. This example demonstrates how this nested example can return valuable information that would be not possible to obtain from an analytical model and would be infeasible for a simulation that fully resolved the flow and solid in the same simulation.

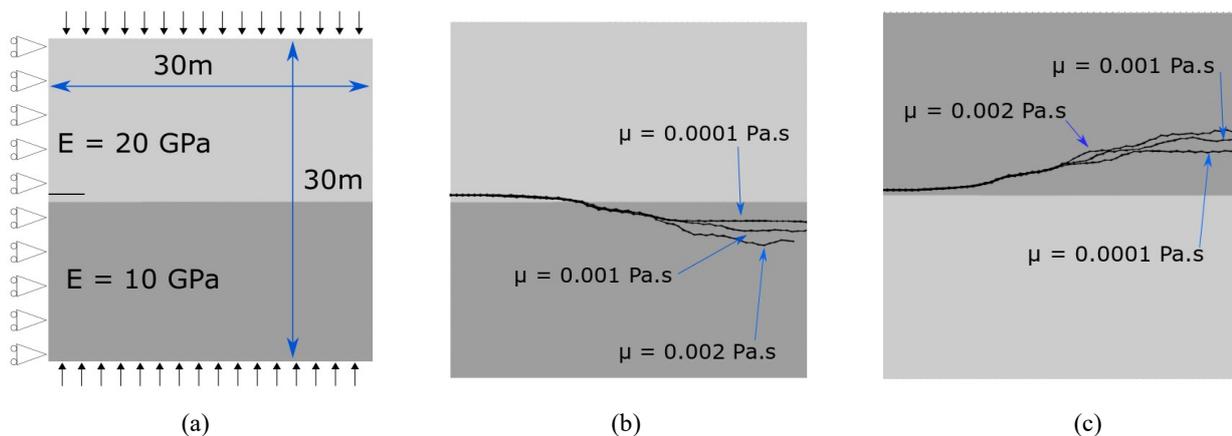


Figure 4 (a) The simulation parameters for the second test example. (b, c) The fracture paths for simulations with different fluid viscosities overlaid onto the elastic medium

## CONCLUSIONS

A method is presented for nesting a lower dimensional MPM simulation onto the manifold provided by an explicit fracture plane of a full MPM simulation to model fluid flow inside fractures. This is a promising method to accurately model the process of hydraulic fracturing. Future work includes implementing the method for 3D fracture sand to allow for interacting with natural fractures.

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