



## Saturated sand column collapse in air: comparison between MPM and coupled DEM-LBM

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### ABSTRACT

Many hazardous natural phenomena like debris flow, avalanches and submerged landslides are governed by the interactions between solid grains and interstitial fluid, and they are still very challenging to simulate numerically. This paper considers a small-scale column collapse experiment, and compares the results of two different numerical approaches: (i) a continuum, two-phase Material Point Method (MPM) and (ii) a Discrete Element Method coupled with the Lattice Boltzmann Method (DEM-LBM). Differences and similarities of the methods are highlighted in order to give an insight into the modelling of saturated granular flows.

**KEY WORDS:** 2-phase double-point MPM; DEM-LBM; column collapse; soil-fluid interaction.

### INTRODUCTION

Many hazardous natural phenomena like debris flow, avalanches and submerged landslides are characterized by rapid movement of a mixture of solid particles and fluid. During motion the two phases can separate. Depending on the flow velocity and the mixture characteristics, the front can be dominated by the solid phase (granular front) or the liquid phase (fluid front). The interactions between these phases governs the propagation of the mixture and this behavior is still poorly understood.

The numerical simulation of these phenomena is attractive, because it allows to use numerical models in the context of hazard assessment and mitigation. One of these methods, the two-phase double-point Material Point Method (2P-DP MPM) has been recently implemented in Anura3D (<http://www.anura3d.com/>, 2016).

The 2P-DP MPM is in this work tested on a laboratory experiment of saturated sand column collapse. In the test, a sample of saturated sand is placed in a box with a movable gate which is rapidly removed, releasing the mixture onto a horizontal plane (Fig. 1). The initial column dimensions are H=6cm, L=4cm and W=5cm. The evolution of the phenomenon, and the relative movement between the phases during the collapse, is recorded with a high-speed camera.

The test, albeit simple, provides a non-trivial benchmark for the continuum model. When the gate is removed, the gravity force induces the development of a failure surface that separates a volume of mobilized material in agitated conditions where the grains interact between each other through brief frictional contacts. Eventually, solid particles can raise in suspension (fluidized state) and be driven by the fluid. During the column collapse, the pore space dilates and contracts quickly, determining strong fluid pore pressure oscillations that cannot be measured with the available laboratory equipment. To give more insight into this behavior, a discrete numerical model is used to reproduce the same experiment. This second model employs the Discrete Element Method for the grains, coupled with the Lattice Boltzmann Method for the ambient fluid (DEM-LBM), and is implemented in the code by Leonardi *et al.* (2016)

The main difference between the two numerical approaches comes in the description of the granular phase, which is treated as an equivalent continuum by MPM, and as a discrete collection of particles by DEM. The use of both methods allows to highlight potentialities and critical points of the two approaches. The DEM-LBM allows to obtain a more realistic distribution of pore pressure, while MPM resolves more correctly the composition of the collapse front.

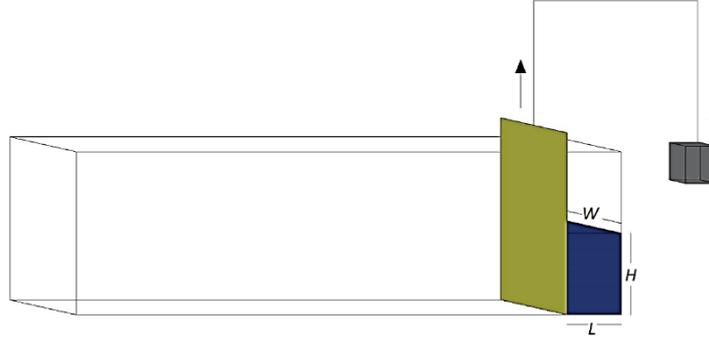


Figure 1 Sketch of the experimental set up for the column collapse test.

## DEM-LBM

DEM-LBM is a hybrid approach where two different simulation strategies are employed for the two phases, and a coupling algorithm tracks the inter-phase transmission of forces.

The granular phase is simulated with DEM as a collection of spherical particles, which interact through collisions and frictional contacts. The particles are allowed a small overlap  $\xi$ , which determines a repulsive force normal to the contact surface, and equal to

$$F_n = k_n \xi + \alpha_n (k_n m)^{1/2} \frac{d\xi}{dt} \quad (1)$$

with  $k_n$  and  $\alpha_n$  the normal contact stiffness and damping, respectively, and  $m$  the particle mass. In the tangential direction, an analogous force is exchanged, modulated by a friction coefficient  $\mu_s$ , and equal to

$$F_t = \max\left(\mu_s F_n, k_t \zeta + \alpha_t (k_t m)^{1/2} \frac{d\zeta}{dt}\right) \quad (2)$$

where  $\zeta$  is the length of a spring connecting the initial point of contact in the particle surface. Additionally, a resistance to rolling has been implemented in order to reduce the spurious effects due to the use of spherical particles. It consists in a moment resisting relative rolling between particles and proportional to the normal load:

$$M_r = \mu_r r F_n \quad (3)$$

where  $\mu_r$  is a coefficient of rolling friction, analogous to  $\mu_s$ . The algorithm is explained in details in Marchelli, Leonardi, & Pirulli (2018).

The particles are immersed in a fluid phase, whose dynamics are solved with LBM. In this method, field variables such as velocity  $v_L$  and pressure  $p_L$  are recovered by tracking the state of a population of fluid molecules, which stream between neighbor nodes of a regular lattice. The conservation of mass and momentum is imposed by a solution of the Boltzmann equation, which drags the molecules dynamics towards Maxwellian equilibrium. The fluid phase has a free interface, whose position is updated using a volume-of-fluid approach.

A key aspect of this approach is that the DEM particles are required to be much larger than the discretization units of the fluid. Therefore, multiple fluid cells are located within each DEM particles. These cells exchange a drag force  $f_d$  with the particles, as:

$$f_d = V_c \rho_L (v_L - v_p) \quad (4)$$

where  $V_c$  is the volume of the fluid cell, and  $v_p$  the particle velocity.

The integration in time is explicit for both methods, but the time-step required by DEM is usually smaller than that required by LBM. The two solvers are then called in a staggered fashion, with multiple DEM iterations in between two consecutive LBM steps. More details about the method, its limitations and possible applications can be found in Leonardi *et al.* (2016).

## TWO-PHASE DOUBLE-POINT MPM

The 2P-DP formulation was initially presented by Bandara (2013) and Wieckowski (2013), and later extended by Martinelli (2015). It assumes that the soil is a superposition of two independent continuum media, hence the solid skeleton and the liquid phase are represented separately by two sets of Lagrangian MPs: solid material points (SMPs) and liquid material points (LMPs). The dynamic behaviour of the continuum can be described with the solid and liquid dynamic momentum balances which are solved at the nodes of the grid. Mass balance equations of the solid phase and the mixture and constitutive relationships are posed at the corresponding MPs in order to update secondary variables.

The force representing the interaction between solid and fluid assumes the expression proposed by Vardoulakis (2004) (Eq. 5)

$$f_{SL} = f_n + f_d = \sigma_L \nabla n_L + f_d \quad (5)$$

$f_n$  accounts for the gradient of porosity, while  $f_d$  (Eq. 6) is a function of the relative movement between solid and fluid. Similarly to the DEM-LBM method, the drag force is a linear function of the relative velocity  $v_L - v_S$ :

$$f_d = \frac{n_L^2 \mu_L}{\kappa_L} (v_L - v_S) \quad (6)$$

The intrinsic permeability  $\kappa_L$  is computed and updated as a function of the effective porosity  $n_L$  and the average grain diameter  $D$  with the Kozeny-Carman formula:

$$\kappa_L = \frac{D^2}{150} n_L^3 / (1 - n_L)^2 \quad (7)$$

Furthermore, the phase transition process in the soil is reproduced by a maximum porosity criterion: below a threshold porosity value ( $n_{\max}$ ) solid-like behavior persists with positive effective stress, updated with constitutive relations typical of soils, like Mohr-Coulomb. On the contrary, above the maximum porosity, grains are supposed to be overly detached; thus the stress transmission is no more possible, the effective stress disappear and the soil behaves like a fluid, with an effective viscosity (Beenakker, 1984), affected by the solid grains presence.

## NUMERICAL MODELS

### Material parameters calibration

Before studying the saturated granular flow, a preliminary calibration of the material parameters governing the behavior of the solid phase is carried out based on dry column collapse tests. The specimen is made of an artificial sand, characterized by a uniform granulometric distribution, mean diameter  $D = 2.5$  mm and grain density  $\rho_s = 2.625$  g/cm<sup>3</sup>. Moreover, the preparation in a loose state, corresponds to an initial porosity value of 0.4.

The DEM model counts 8008 particles initially placed in a column with the same dimensions of the experiment. Boundary walls have frictional properties when interacting with the solid phase. Particles can slip on the surface only when maximum friction is overcome. The same frictional coefficients are assumed both for the particle-particle contact and the wall-particle contact and are calibrated by comparing the numerical and experimental final shape of the deposit. The restitution coefficient has been taken from similar tests (Marchelli et al, 2018).

The MPM model neglects the frictional effect of the lateral boundary and applies a reduced thickness of the model (0.5 cm). The bottom boundary is fully fixed, while roller boundary conditions are applied at the other surfaces. A linear-elastic perfectly plastic model with Mohr-Coulomb failure criterion is used.

A series of parametric analyses has been performed to calibrate the macroscopic friction angle. Table 1 lists the set of parameters for both DEM and MPM models that give the best fit of the experimental results in terms of final shape of the deposit as shown in Figure 2.

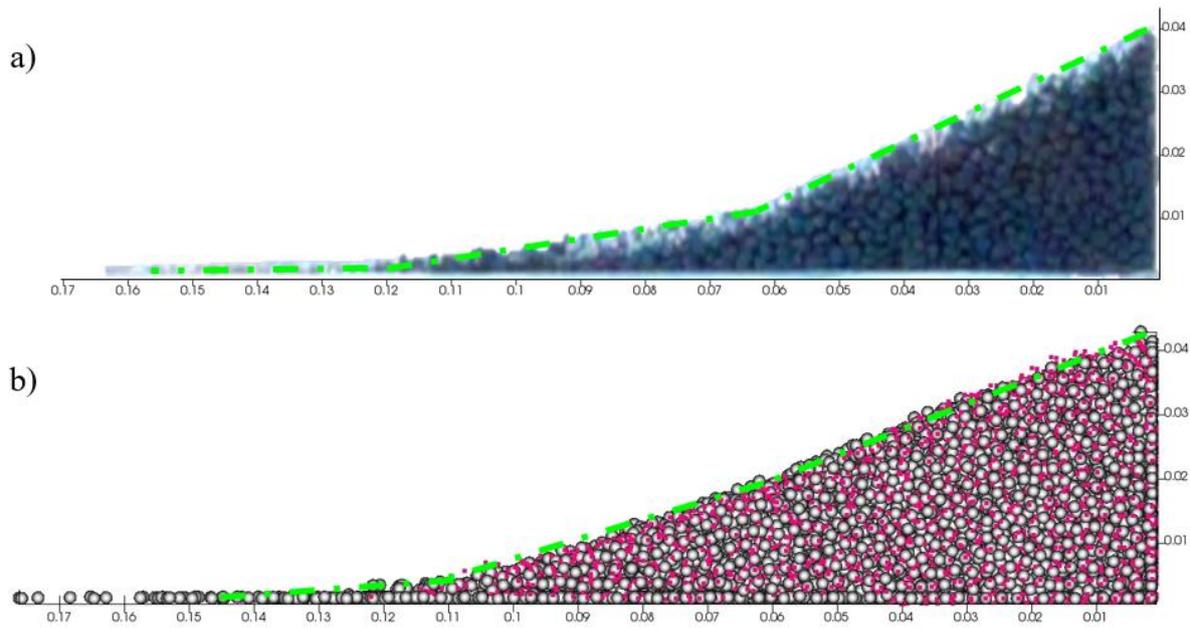


Figure 2 Final configuration dry column collapse: a) experimental result b) numerical results where red dots stands for MPM and grey spheres for DEM.

Table 1-Best-fit DEM and MPM-solid phase parameters.

<i>DEM</i>		<i>MPM</i>	
Density (kg/m <sup>3</sup> )	2600	Density (kg/m <sup>3</sup> )	2600
Normal contact stiffness (N/m)	$0.2 \cdot 10^4$	Initial effective porosity	0.4
Tangential contact stiffness (N/m)	$0.057 \cdot 10^4$	Effective Poisson ratio	0.3
Damping	0.04	Young Modulus (kPa)	$10^4$
Restitution	0.88	Effective cohesion (kPa)	0.0
Friction coefficient	0.577	Effective friction angle (°)	35
Rolling coefficient	0.05		

### Saturated soil models

Concerning the liquid phase, pure water has been considered and the material behavior is described with a simple Newtonian model. Both methods have been extensively tested in the classical dam break problem showing good agreement with experimental results and other numerical methods (Janßen and Krafczyk, 2011; Zhao, Liang and Martinelli, 2017).

If on the one hand the dynamic viscosity is the same for both model, and equal to  $10^{-6}$  kPa·s, on the other side the compressibility of the fluid is completely different: for the DEM-LBM model, the fluid is considered almost incompressible, while for the MPM model the compressibility is defined by the bulk modulus. In MPM a reduced value of the bulk modulus ( $2 \cdot 10^4$  kPa) compared to pure water is used to speed up the calculation. A cavitation threshold is imposed in MPM to overcome numerical difficulties, thus only positive pressures are allowed. To improve stability, a turbulence model has been activated in LBM, conceptually identical to the large-eddy technique used in Leonardi *et al.* (2018).

After the calibration of the material parameters based on the dry case, the saturated mixture behavior is explored during propagation in air. The geometrical features and the material parameters are those introduced in the previous section.

The MPM model applies a structured mesh; 4 LMPs and 4 SMPs are assigned to each initially active element. All calibrated material parameters, as explained in the previous paragraph, are maintained. A mean diameter of 2.5 mm is used to update the intrinsic permeability and the drag force, with Eq. 6 and 7. The boundary conditions for the liquid are identical to the one of the solid.

The coupled DEM-LBM model applies a “no slip” condition for the fluid phase at all boundaries. The initial sample is assembled by gravity-induced deposition inside the release tank, always in dry conditions. The fluid is discretized using a uniform grid spacing of 0.6mm in every direction.

## RESULTS

The experimental sample is prepared in a loose state by slowly pouring the wet sand inside the water with a small spoon and then adjusting the level of water with a syringe. After gate opening, the water tends to flow out of the sample very rapidly and the top part of the column desaturates during collapse. However, it is not possible to distinguish a clear separation between the phases because a small amount of water remains attached to the grains by surface tension and it influences the movement of the top part of the column. The fluid front seems to be slightly ahead of the granular front during the first milliseconds. The maximum runout is reached in about 0.5 s and it is 12 cm (Fig. 3).

Figure 4 compares the results obtained with the two numerical models. In the DEM-LBM model the collapse develops quite slowly and the formation of a granular front is recognizable up to 0.5s, then the water slowly flows out of the sand. If a few grains that separates from the bulk flow running away are neglected, the maximum runout of 13 cm is reached in about 0.55s. The MPM model predicts a faster collapse, with higher velocity of the front. The maximum runout is reached in 0.3s and it is approximately 15 cm. There is no clear separation between granular and liquid front. The final shape of the solid deposit in MPM is approximately a straight line, while in DEM-LBM it is convex (Fig. 5). The discrepancies with the experimental results are mainly due to the behavior of the top part of the column which is in partially saturated conditions and not dry as assumed by the numerical models.

Figure 6 shows the evolution of the total kinetic energy of the system (solid+fluid) normalized by the initial total potential energy obtained with the two numerical models in both dry and saturated conditions. In dry conditions the two models give very similar results; in contrast, very different curves are obtained in saturated conditions. The peak of kinetic energy in MPM is much higher than in DEM-LBM and it occurs earlier (0.12 s instead of 0.18 s). This confirms, as previously observed, that in MPM the column collapses much faster than in DEM. Compared to the dry case, in MPM the maximum kinetic energy increases, while in DEM it decreases.

Since the results for the one-phase simulation are in good agreement, the differences must be related to the solid-fluid interaction model. A significant source of difference lies in the liquid pressure distribution. DEM-LBM gives a relatively smooth pressure distribution and both positive (compression) and negative (suction) pressures are computed. Negative pressures increase the interaction forces between grains slowing down the failure. In MPM significant oscillations are observed in the pore pressure and only positive (compression) values are allowed since the cavitation threshold is set to 0 kPa. The use of a large cavitation threshold leads to numerical difficulties and the calculation crashes. Neither the use of pressure smoothing nor strain smoothing techniques improves the results. Future work is required to address this problem.

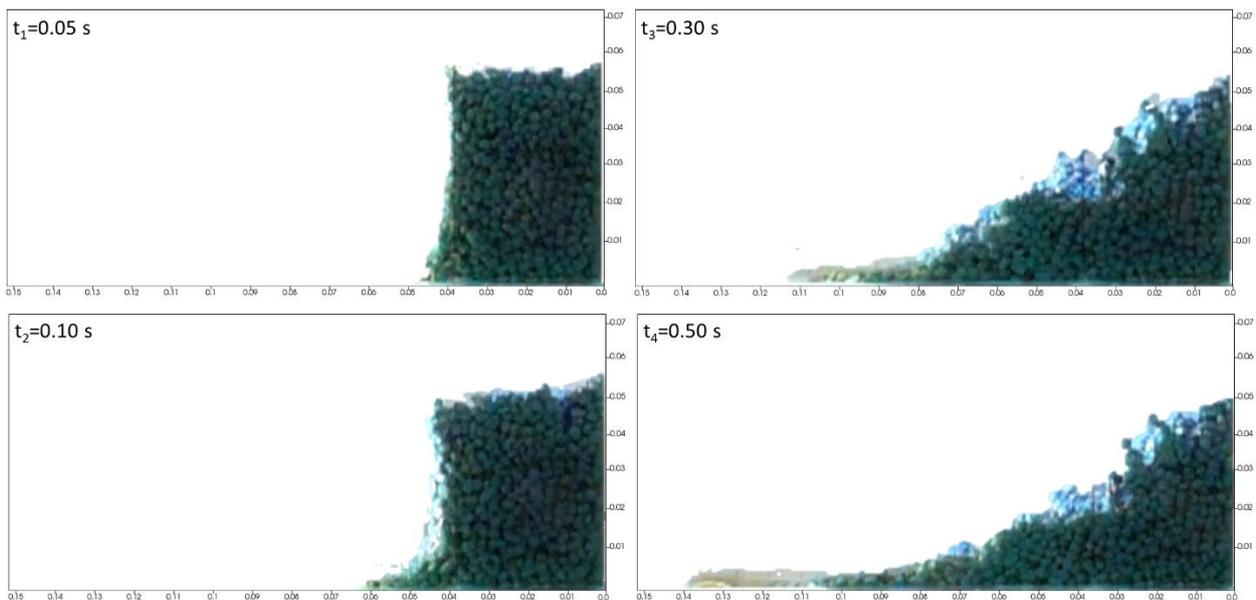


Figure 3 Photos of the saturated soil collapse experiment in four different time instants.

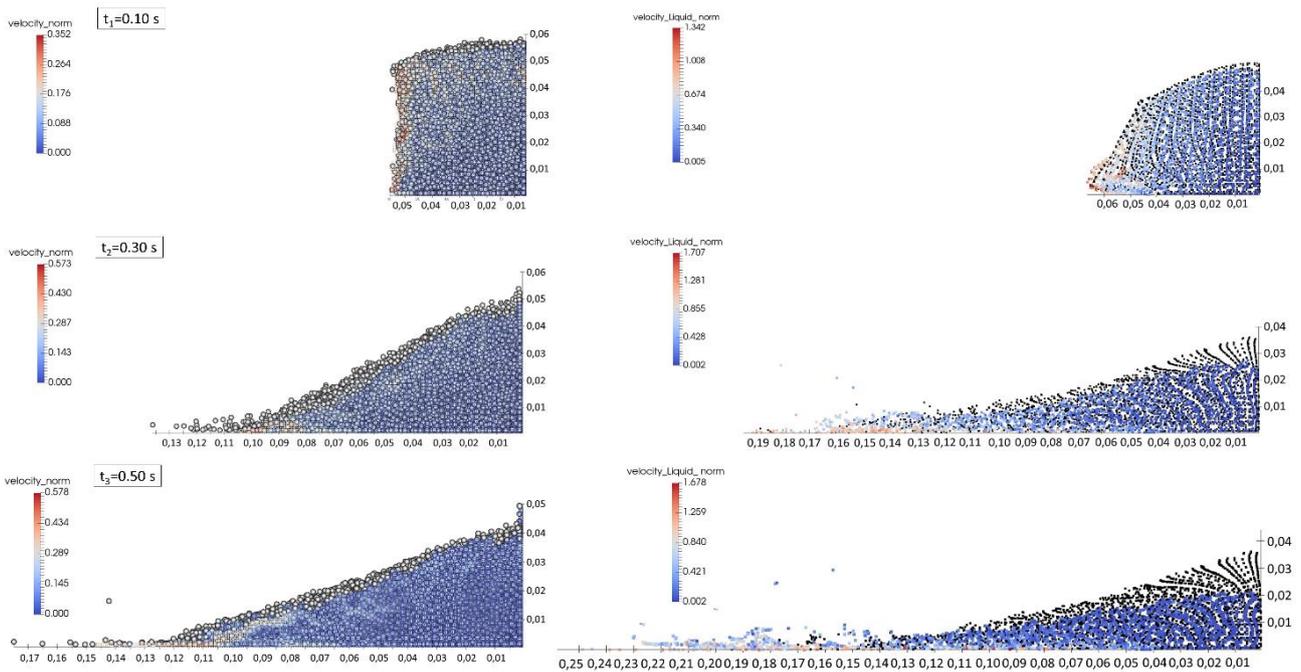


Figure 4 Numerical results at different time instant obtained with DEM-LBM (left) and MPM (right). Grey circles indicate the position of the solid phase and the blue-red colour scale indicate the liquid velocity.

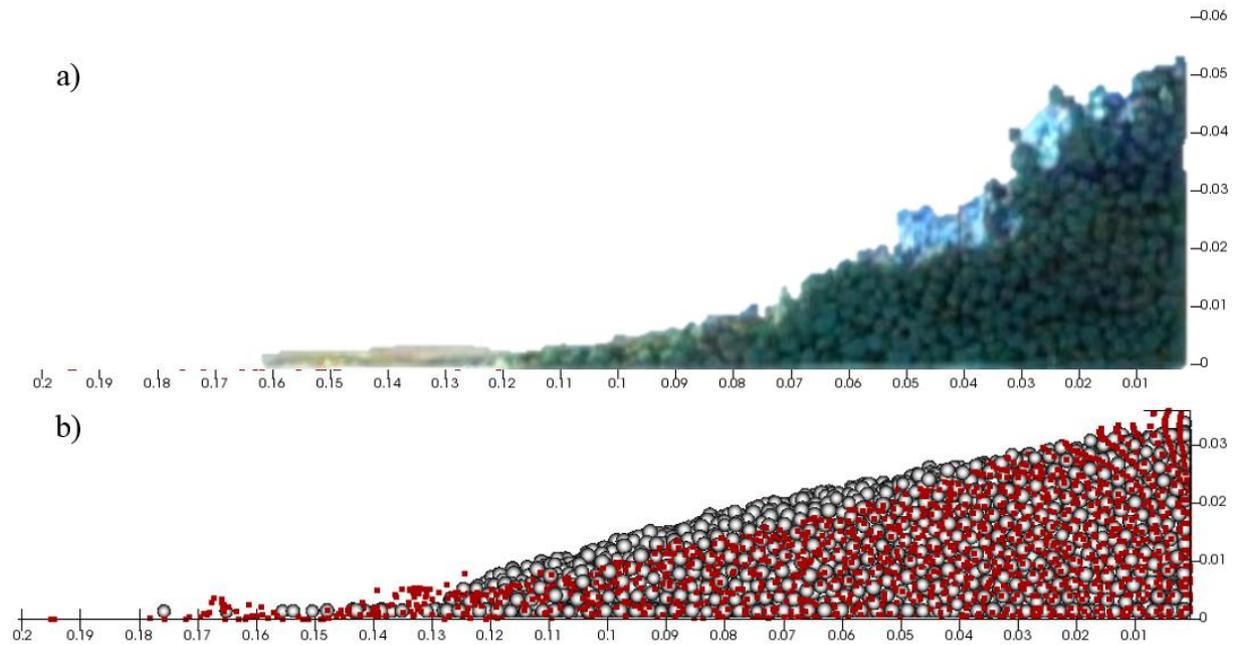


Figure 5 Final configuration saturated column collapse: a) experimental result b) numerical results where red dots stands for MPM and grey spheres for DEM.

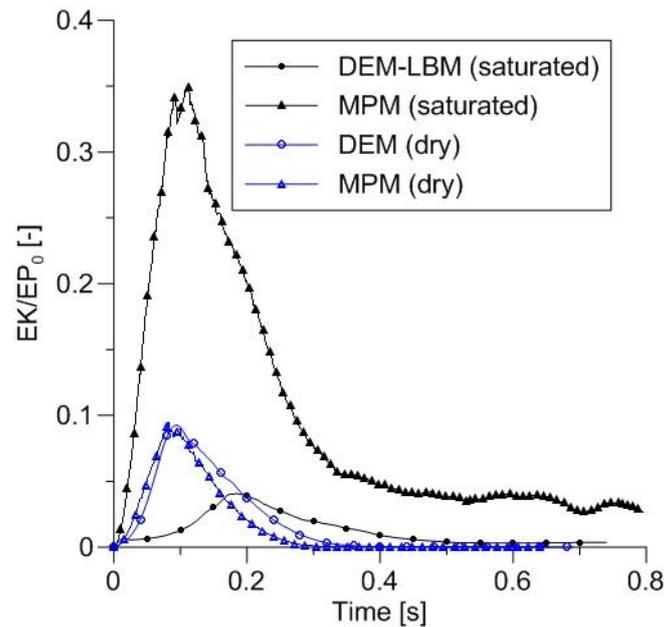


Figure 5 Numerical normalized kinetic energy of the system for the dry and saturated column collapse.

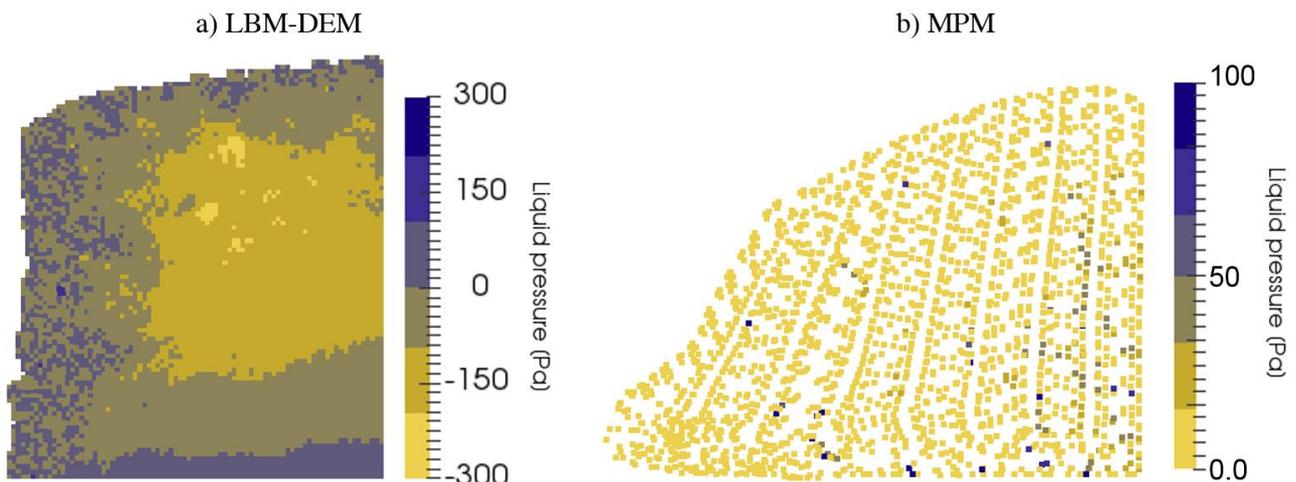


Figure 6 Liquid pressure at  $t=0.1$ s computed with a) LBM-DEM and b) MPM (compression is positive). Solid phase is not displayed.

## CONCLUSIONS

DEM-LBM and 2P-DP MPM are advanced numerical models that can simulate solid-fluid interactions in granular materials including separations between the constituents. DEM-LBM applies a micromechanical approach to simulate grain-grain interactions and a very fine discretization for the fluid phase is necessary. While the discrete approach offers more insight into the microscopic structure of the grains, it is also more computationally demanding, with the algorithm cost growing with the total number of particles, limiting the simulations to relatively large grains. On the other hand, the efficiency of MPM is not altered by the grain size.

2P-DP MPM applies a continuum approach, which arises questions on the modelling of the fluidization process and constitutive model of the constituents. These aspects should be further investigated in the future. Currently, simple elastoplastic models are used for the solid and the criteria of maximum porosity is applied for the solid-fluid transition with reasonably good results. The method is easily applicable to real-scale problems.

This paper compares the results obtained in simulating a saturated sand column collapse in air. The MPM model predicts a faster collapse with higher velocities and kinetic energy compared to DEM-LBM; this could be possibly related to the oscillations in the pressure distribution observed in MPM and the use of a cavitation threshold equal to 0.

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