

Calibration of hypoplastic parameters using an MPM model of a CPT

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ABSTRACT

MPM can be used to model cone penetration tests (CPT). This study aims at showing how inverse analysis can be employed to calibrate the MPM model of a CPT, and in particular the input parameters of an advanced constitutive model, using field data. The considered CPT test was carried out in sandy soil. The selected constitutive model belongs to the class of hypoplasticity. A gradient-based optimization algorithm is used to simultaneously calibrate 5 of the 13 hypoplastic input parameters. The inverse analysis of the MPM model successfully converged after few iterations.

KEY WORDS: Inverse modelling; hypoplasticity; cone penetration test.

INTRODUCTION

Cone penetration tests always produce large displacements, rotations and deformations of soil elements, as well as a complex response of the soil to the displacements imposed by the penetration process (e.g. Arshad et al., 2014). In recent years, the numerical simulation of this mechanism has been the subject of many research contributions (Gens et al., 2016), including MPM applications (Beuth, 2012; Kafaji, 2013; Ceccato et al., 2016; Galavi et al. 2018; Ghasemi et al., 2018). The main aims of this study are: confirming the effectiveness of MPM in simulating a CPT; showing the possibility to calibrate the parameters of an advanced constitutive model using CPT field data.

MATERIALS AND METHODS

Experimental data

The data used in this study come from a confidential database from Fugro related to the Brussels Wind farm II Project (Fugro Report N6016/04). The data refer to a offshore sandy formation, known as Tongeren sand, and in particular to a CPT carried out 10 m below the seabed (Figure 1). Results from three isotropically-consolidated drained triaxial tests (Table 1), conducted on samples collected 6 m below the seabed in a nearby borehole, have also been used to determine the initial values of the soil model parameters (Table 2). Index tests results indicate that the minimum and maximum void ratios of the soil are equal to 0.35 and 1.37, respectively. The void ratio of the material on site, obtained in the laboratory on an intact sample extracted from the borehole in which the CPT was conducted, is 0.62. Figure 1 shows that the CPT tip resistance reaches a steady state condition after about 40 cm of cone penetration. Therefore, the simulation of the initial 60 cm of penetration, from 10.0 to 10.6 m below the ground surface, are deemed to be sufficient to retrieve the properties of the investigated soil. The CPT profile also shows a distinct increase of tip resistance at about 15 cm of penetration, which does not seem to be coherent with the hypothesis of a unique homogeneous layer throughout the investigated depths. Accordingly, only the experimental data which represent the steady state tip resistance are going to be employed for model calibration, i.e. the 15 observations used for the inverse analysis only refer to penetration depths between 40 cm and 60 cm.

Table 1 Triaxial tests on samples extracted from borehole BH-WFS2-5

Test number	Initial void ratio	Confining pressure (kPa)
1	0.57	190
2	0.52	380
3	0.49	760

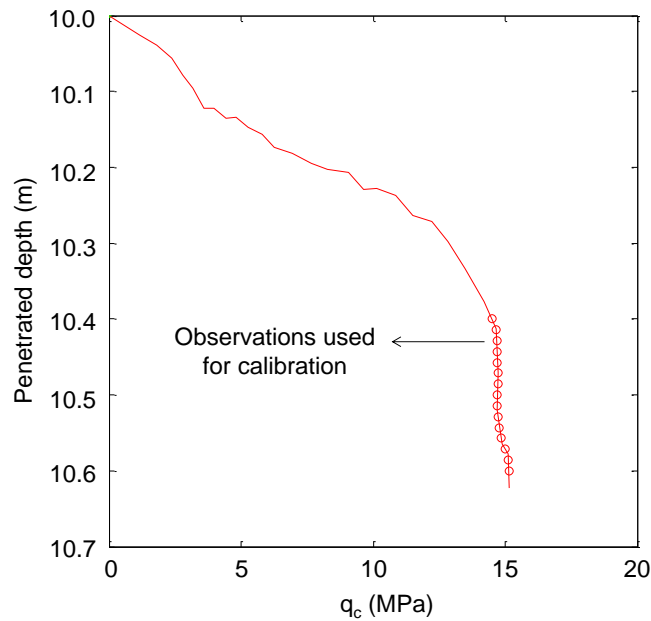


Figure 1 CPT results and observations used to calibrate the soil model

Hypoplastic model

A currently-considered standard hypoplastic model for granular materials is that of Gudehus (2004). It includes 8 main material parameters, plus other 5 so-called intergranular parameters introduced by Niemus and Herle (1997) to eliminate ratcheting—i.e. excessive accumulation of deformation predicted for small stress cycles—and to improve the model performance in cyclic loading. Typical ranges of the considered hypoplastic model parameters are reported in von Wolffersdorff (1996). Table 2 shows the meaning of the model parameters, the values adopted in the initial MPM simulation of the CPT, which are based on the results of the three triaxial tests, and the indication on whether they were also calibrated by inverse analysis considering the CPT experimental data.

Table 2 Input parameters of the adopted hypoplastic model

Parameter	Meaning	Initial value	Calibrated
ϕ_c	Critical friction angle	31°	no
e_{io}	Maximum void ratio at zero pressure	0.37	no
e_{d0}	Minimum void ratio at zero pressure	1.35	no
e_{c0}	Critical void ratio at zero pressure	1	Yes
α	Controls peak friction and dilative behavior	0.012	Yes
β	Controls the influence of void ratio on incremental stiffness	0.14	Yes
h_s	Granular stiffness	1.5E+5 kPa	Yes
n	Controls the stiffness dependency from mean pressure	0.24	Yes
β_r	Controls the stiffness degradation from small to large strain	0.4	no
R_{max}	Strain range in which the stiffness is linear	0.0002	no
M_f	Coefficient by which the stiffness is increased upon 180 strain reversal	1.5	no
M_t	Coefficient by which the stiffness is increased upon 90 strain reversal	1.2	no
χ	Controls the stiffness degradation from medium to large strain	1	no

MPM model

Figure 2 shows the MPM schematization adopted to simulate the CPT boundary value problem. The background mesh has a total of 9936 elements including the initially inactive elements (in grey in the Figure). The effect of mesh size was not analyzed in this study. The inactive elements are activated during the calculation if material points move into the space they occupy. Triangular elements with linear interpolation of the displacements are used. The number of material points is 105120. The MPM moving mesh concept is adopted in all simulations (Kafaji, 2013). The numerical simulations are performed using an axisymmetric geometry. To avoid boundary effects, the side boundary is placed at a distance of 30 D from the symmetry line (the diameter of the cone D is

3.57 cm). This space is divided in three parts with mesh sizes getting larger and number of particles per elements decreasing as the distance from the symmetry line increases. Displacements at the side boundary are constrained in the radial direction and free in the vertical direction. The bottom boundary, placed at a distance of 30 D from the initial position of the cone tip, is fully fixed. The simulations are performed considering a submerged one-phase material in drained conditions. Stresses in the soil are initialized using a typical K0-procedure. Considering that the 10 m thick soil located above the cone does not significantly interact with the shaft yet it affects the initial stresses in the domain around the cone, it is modelled considering a 10 D thick (36 cm) layer of hypoplastic material overlaid by a 10 cm thick layer of elastic material with the following properties: Young modulus equal to 1000 kPa; Poisson ratio equal to 0.0; and very high density proportional to the ratio between the real thicknesses of the soil layer above the cone and the modelled one. In this case, a material density of 887.8 kN/m³ is needed to simulate 9.64 m of submerged sand imposing an effective stress at the bottom of the elastic material equal to 92 kPa. The cone penetrometer “pile” is modelled as a rigid body penetrating into the soil with a prescribed velocity equal to 0.02 m/s, a value common in standard practice. A contact algorithm is used to model the frictional contact between the pile and the soil. The adopted friction angle value (11°), is representative of a sand to polished steel contact (Murray and Geddes, 1987). Considering the above conditions, the time needed to run one model simulation is approximatively equal to 8 h.

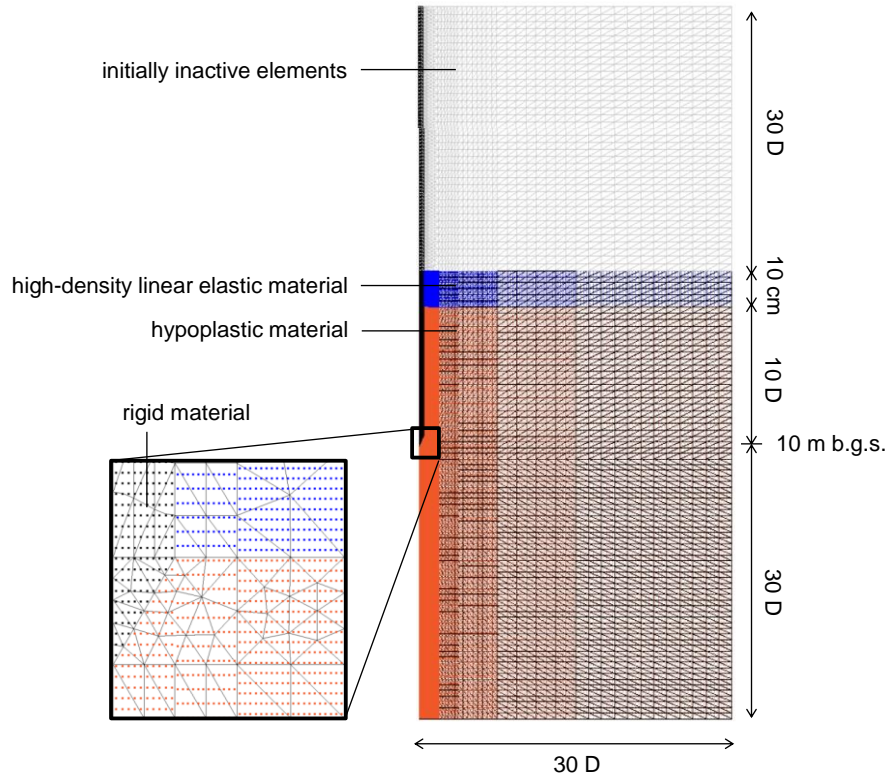


Figure 2 Scheme of the MPM model of CPT.

Inverse analysis

A gradient-based non-linear regression analysis is used to calibrate 5 of the 13 input parameters of the adopted hypoplastic law. The regression was conducted using UCODE (Poeter and Hill, 1997), a model independent algorithm designed to allow inverse modeling posed as a parameter estimation problem. The weighted least-squares objective function $S(\underline{b})$ minimized during the inverse analysis is expressed by:

$$S(\underline{b}) = [\underline{y} - \underline{y}'(\underline{b})]^T \underline{\omega} [\underline{y} - \underline{y}'(\underline{b})] = \underline{e}^T \underline{\omega} \underline{e} \quad (1)$$

where: \underline{b} is the vector of the 5 input parameters being simultaneously estimated; \underline{y} is the vector of the 15 observations being matched by the regression (see Figure 1); $\underline{y}'(\underline{b})$ is the vector of the corresponding computed

values; $\underline{\omega}$ is the weight matrix, being the weight of every observation taken as the inverse of its error variance, herein assumed equal to 0.01; and \underline{e} is the vector of residuals.

The regression implies, at any given iteration, multiple runs of the numerical model to update the chosen input parameters. To this aim, a sensitivity matrix employed is computed using a perturbation method and a forward difference scheme. The elements of the sensitivity matrix, X_{ij} , are computed as follows:

$$X_{ij} = \partial y_i / \partial b_j \quad (2)$$

where: y_i is i -th observation; b_j is j -th input parameter.

It means that, within each iteration N simulations of MPM are required when N is the number of calibrated parameters, herein equal to 5. As mentioned before, each CPT forward model takes about 8 hours. To reduce at a minimum the time required for the regression, all of perturbed simulations are executed in parallel at each iteration. Two convergence criteria are used to conclude the optimisation: (i) maximum parameter change lower than a user-defined percentage of the parameter value at the previous iteration, herein equal to 5%; or (ii) objective function change lower than a user-defined amount for three consecutive iterations, herein equal to 0.05.

RESULTS

Model calibration

The initial values of the 13 hypoplastic model parameters were estimated, as already mentioned, by curve fitting the stress strain response of three drained triaxial compression tests. More details on the optimization procedure adopted to determine the initial values of the parameters are reported in Cuomo et al. (2018). Five of these parameters (see Table 1) have then been further calibrated to minimize the fit between CPT experimental data and MPM model results, using the procedure described in the previous section. Figure 3 show that the calibrated model, differently from the initial model, adequately simulates the final tip resistance of the CPT. Figure 4 shows the values of the 5 calibrated parameters, as well as the value of the objective function defined in Eq. 1, at each iterations of the regression. The calibration procedure converged after only 7 iterations, when the regression convergence criterion is satisfied. The lowest value of the objective function, almost three orders of magnitude lower than its initial value, is reached at iteration 4. The results indicate that the optimal values of the five calibrated parameters (see also Table 3) are always higher than their initial values. The parameter undergoing the highest variation from its initial estimate is the granular stiffness, h_s .

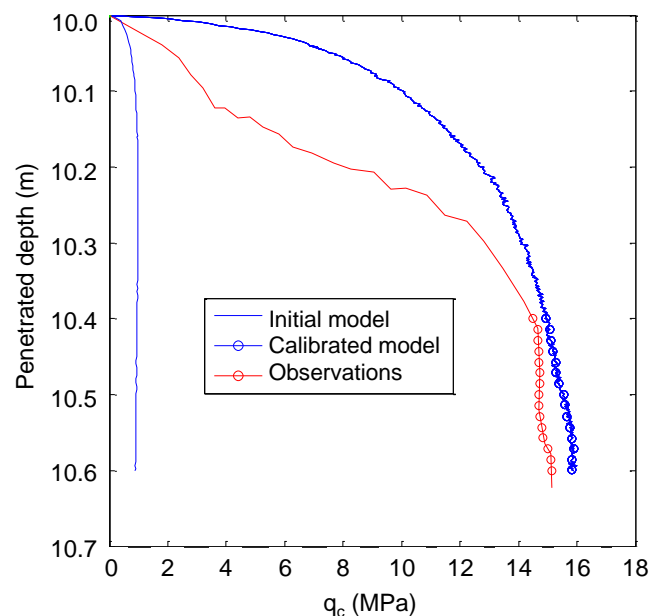


Figure 3 Comparison between CPT data and computed tip resistance for initial and calibrated values of the input parameters

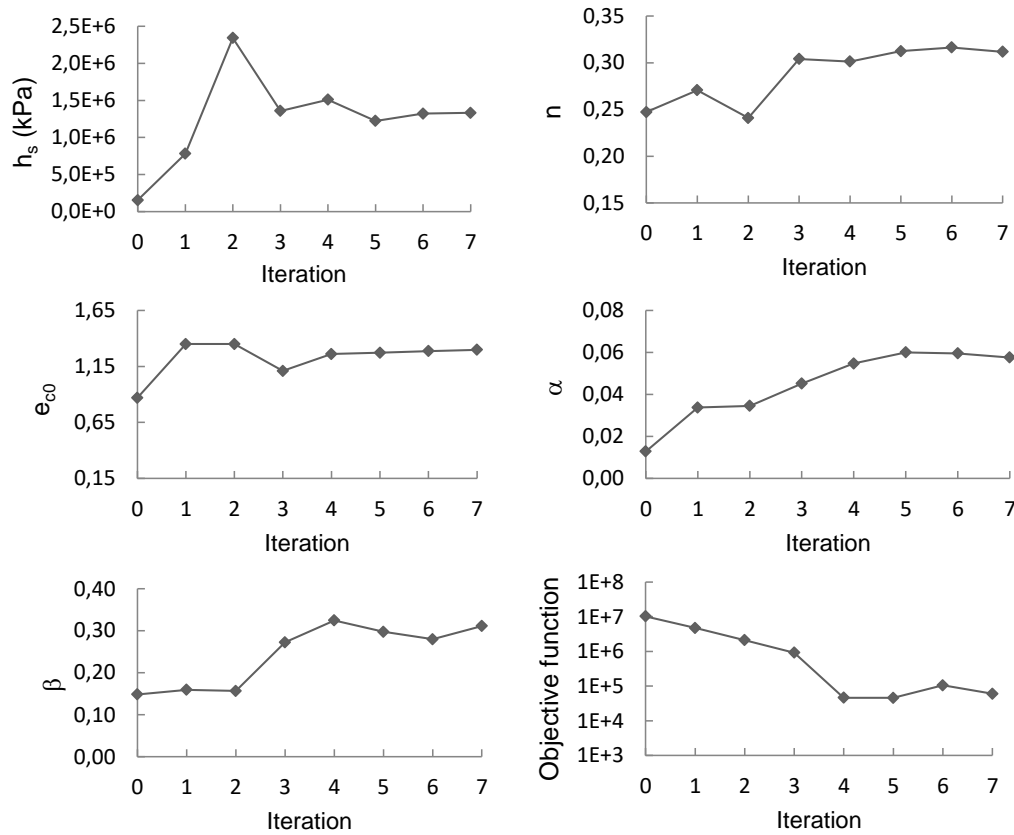


Figure 4 Results of regression at each iteration: values of parameters and objective function

Discussion

Figure 5 illustrates the direction of the particle movements at the end of penetration, for both the initial and calibrated MPM simulations. As expected, the particles close to the cone are always moving downwards, yet the shape of the lump of soil accompanying the cone varies significantly in the two cases, being wider in the calibrated model. The latter model is also characterized by a significantly larger area around the cone affected by outward horizontal displacements as well as by smaller deformations in the upper part of the soil. These differences could be attributed to the fact that the set of parameters adopted in the two cases (Table 3) relate to a contractive behavior upon shearing, for the initial simulation, and to a dilative behavior, for the calibrated simulation. This can be easily seen at representative elementary volume scale, for instance by looking at the simulations of the three triaxial test from Table 1 (Figure 6). In fact, the comparison between the experimental data and the model results indicate that the calibrated values of the input parameters do not adequately reproduce the tests. However, this is not surprising because the void ratio of the soil samples tested in the laboratory is lower than the void ratio of the sand tested in-situ with the CPT.

A final check on the significance of the set of calibrated parameters for the MPM simulation of the CPT is performed by means of a parametric analysis (Table 3). Five simulations are considered in which the base case is the initial simulation and the five hypoplastic parameters are then individually changed, one by one, starting from their calibrated values. Figure 7 shows the comparison between observed and computed tip resistance for the 5 simulations of the parametric analysis. Somewhat unexpectedly, only two of the five parameters (h_s and α) produce results that differ from the base case. The results, besides showing that 2 parameters are more important than the other three, seem to suggest a strong correlation among the input parameters, i.e. cross-dependency effect, and a markedly non-linear behavior of the MPM model in reproducing the CPT tip resistance. In other words, they highlight the importance of the simultaneous calibration of all the considered parameters and, therefore, the usefulness of adopting, to this purpose, an automated inverse analysis algorithm rather than a trial-and-error calibration approach.

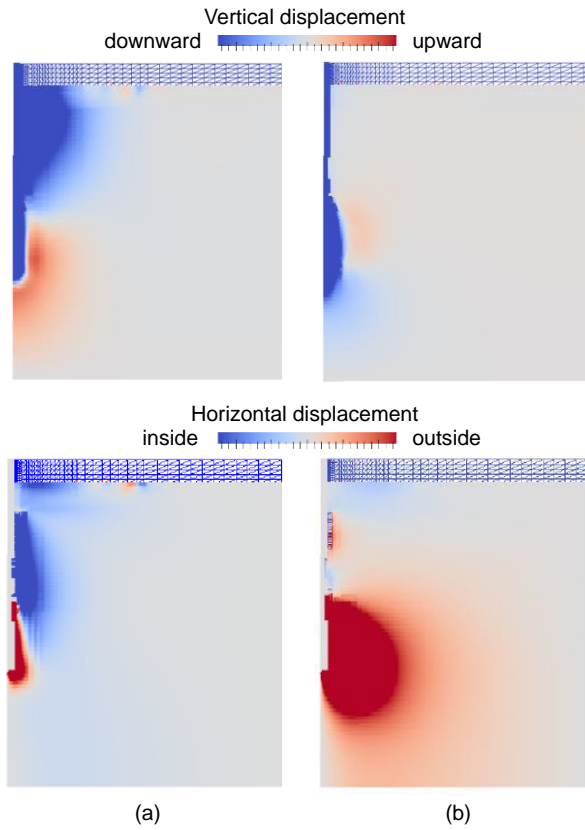


Figure 5 MPM results: direction of horizontal and vertical displacement at end of penetration for the initial (a) and calibrated models (b)

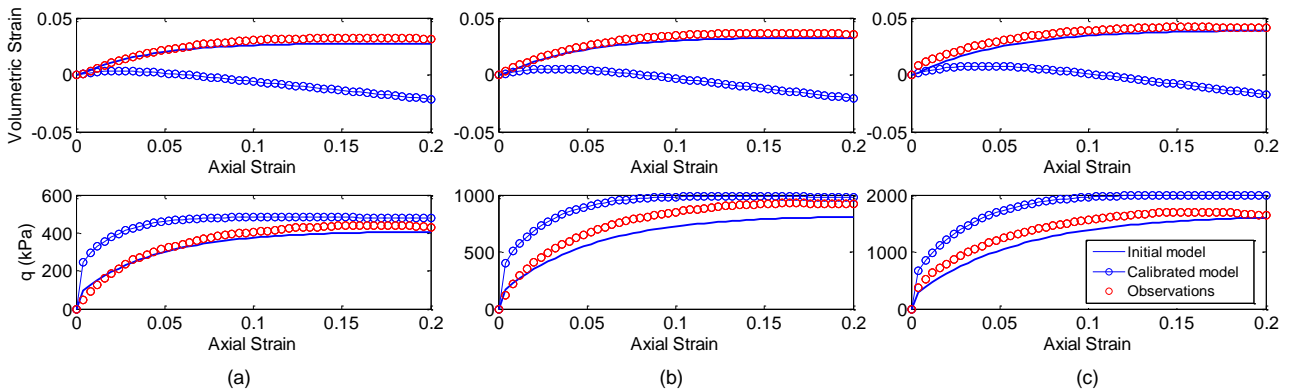


Figure 6 Comparison between experimental data from triaxial tests and hypoplastic model results for the initial and calibrated values of the input parameters

Table 3 Values of the hypoplastic model parameters in the parametric analysis

Parameter	Initial values	Calibrated values	Simulation A	Simulation B	Simulation C	Simulation D	Simulation E
h_s	1.5E+5	1.5E+6	1.5E+6	1.5E+5	1.5E+5	1.5E+5	1.5E+5
n	0.25	0.30	0.25	0.30	0.25	0.25	0.25
e_{c0}	0.87	1.26	0.87	0.87	1.26	0.87	0.87
α	0.01	0.05	0.01	0.01	0.01	0.05	0.01
β	0.15	0.32	0.15	0.15	0.15	0.15	0.32

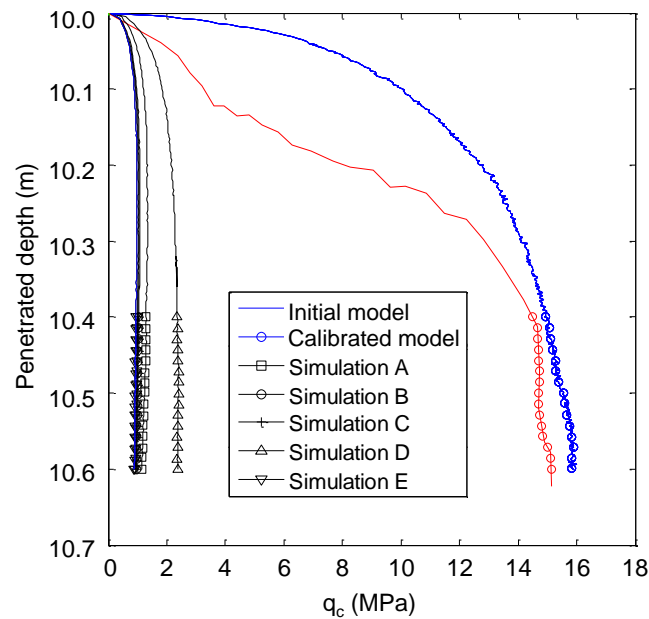


Figure 7 Comparison between CPT observations and computed tip resistance for the 5 simulations of the parametric analysis

CONCLUSIONS

The main results of this study confirm the effectiveness of MPM in simulating CPT in sandy soils. They also highlight the importance of correctly calibrating the input parameters of the constitutive model adopted to simulate the material behavior in order to adequately reproduce CPT experimental data.

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