



MPM Simulation of fine particle migration process within unsaturated soils

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ABSTRACT

Fine particle migration process induced by seepage forces is considered as one of the main mechanisms leading to failures of earth dams and colluvial soil slopes. So far, this process has been mainly simulated with the Finite Element Method (FEM). However, large soil deformation may take place at and after the failure, which is beyond the capability of FEM. In this paper, a coupled seepage-erosion formulation suitable for the Material Point Method (MPM) is proposed for this process based on the mixture theory. By simulating the fine particle migration process within both saturated and unsaturated 1D soil column under water pressure gradient, the formulation capabilities as well as its MPM implementation are validated.

KEY WORDS: internal erosion; fine particle migration; unsaturated soil; material point method

INTRODUCTION

Soils in earth dams or colluvial slopes are usually composed of mixed coarse and fine particles, which are susceptible to seepage-induced internal erosion. With seepage of sufficient velocity, fine particles in these broadly graded or gap-graded soils may be detached from the soil structure by the interstitial flow and transported with the seeping water within the pores formed by the coarse matrix. This fine particle migration process will inevitably alter the local distribution of the size and content of fines in soils, lead to local variations of hydraulic and mechanical characteristics of soils, and may destabilize the soil structure. Traditionally, the fine particle migration process has been mainly simulated with the Finite Element Method. These simulations concentrate on the evolution of internal erosion process and the associated hydro-mechanical responses within soils prior to failure. However, large soil deformations usually take place at and after the failure, which is beyond the capability of FEM simulation. Therefore, this paper simulates the fine particle migration process with the Material Point Method—a promising mesh-free method suitable for large deformation problems. The paper proposes a coupled seepage-erosion formulation suitable for MPM based on the mixture theory and implement it into an explicit MPM code. Sets of internal erosion tests performed on saturated soil columns, as well as the infiltration induced fine particle migration process within an unsaturated soil column, have been simulated with satisfying accuracy by the coupled seepage-erosion MPM code.

GOVERNING EQUATIONS

Soils susceptible to seepage-induced internal erosion are usually unsaturated soils composed of mixed coarse and fine particles with the pore spaces partially filled with fluid. To describe the fine particle migration process mathematically, the soil is treated as a three-phase multi-species porous medium based on the mixture theory (Coussy, 2004). As illustrated in Figure 1, this conceptualized porous medium consist of: solid phase S , liquid phase L , and gaseous phase G , with their respective species shown by the following sets:

$$S = \{c', e'\}; L = \{w, e\}; G = \{g\} \quad (1)$$

In this way, the coarse and fine particles in the soil matrix of soils (phase S) are represented by c' and e' species respectively; the liquid phase L consists of both water species w and fluidized fines species e ; whereas, the gaseous phase G is assumed contain only the dry air species g . The fine particle erosion process is represented by the phase transfer process from the solid fines species e' to the fines species e in the phase L ; the fluidized fines species e will also be transported by the flowing water as its suspension.

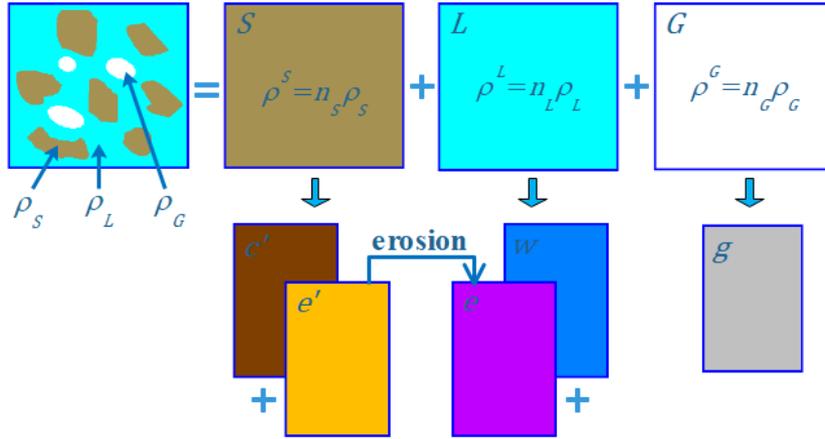


Figure 1 Representation of an unsaturated erodible soil as a superimposed mixture: (top) three-phase macroscopic model of the REV; (bottom) multiple-species macroscopic model of the REV (after Lei et al, 2014)

Mass conservation for fine species

By privileging the solid phase, the mass conservation equation for fine particle species can be written as follows (Lei et al., 2017a, b):

$$S_L n \frac{\partial}{\partial t} c_e + \mathbf{J}^L \nabla \cdot c_e + \nabla \cdot \mathbf{J}_d^e + \frac{1-c_e}{\rho_e} \frac{\partial}{\partial t} m_e = 0 \quad (2)$$

In which, n is the porosity, S_L is the liquid saturation, c_e is the volumetric concentration of fine particles; ρ_e and m_e are the intrinsic density and the mass fraction of solid fines respectively; \mathbf{J}^L is the volume flux of liquid phase $\mathbf{J}^L = n S_L (\mathbf{v}^L - \mathbf{v}^S)$, with \mathbf{v}^L and \mathbf{v}^S being the velocity of liquid phase and solid phase respectively; \mathbf{J}_d^e is the diffusion flux of the fines species $\mathbf{J}_d^e \approx -D \nabla c_e$, which will be neglected in the following simulations.

Momentum balance for liquid phase

Since the momentum change due to fines exchange is small, the momentum balance equation for the liquid phase is assumed can be adopted (Bandara & Soga, 2015):

$$\rho_L \mathbf{a}^L = -\nabla p_L - S_L n \frac{\rho_L \mathbf{g}}{k_L} (\mathbf{v}^L - \mathbf{v}^S) + \rho_L \mathbf{b} \quad (3)$$

In which, \mathbf{a}^L is the acceleration of liquid; ρ_L is the intrinsic density of liquid phase; k_L is the soil hydraulic conductivity; \mathbf{g} and \mathbf{b} are the gravity and body force respectively.

Erosion law

The variation of fines content in the soil skeleton is assumed to be due only to the suffusion mechanism. Following Lei et al. (2017a), Cividini & Giada (2004)'s erosion law is chosen to calculate the fines variation rate at pore surfaces:

$$\frac{\partial}{\partial t} \frac{m_e}{\rho_e} = -k_e \cdot (x_e - x_{e^*}) \cdot \|\mathbf{v}^{LS}\| \quad (4)$$

with

$$x_{e^*} = x_{e^*0} - (x_{e^*0} - x_{e^*}) \frac{\mathbf{v}^{LS}}{\mathbf{v}^*} \quad \text{if } 0 \leq \mathbf{v}^{LS} \leq \mathbf{v}^*$$

$$x_{e^*} = x_{e^*}^* - \alpha_{er} \cdot \log \left(\frac{\mathbf{v}^{LS}}{\mathbf{v}^*} \right) \quad \text{if } \mathbf{v}^{LS} > \mathbf{v}^*$$

In which, $x_{e'}$ and $x_{e'\infty}$ are the current and the ultimate (long term) mass fractions of erodible fines in the solid skeleton, respectively; $\|\mathbf{v}^{LS}\|$ is the norm of the relative velocity vector of pore water to soil skeleton ($\mathbf{v}^{LS} = \mathbf{v}^L - \mathbf{v}^S$); v^* is a reference low flow velocity, under which the variation of $x_{e'\infty}$ with v^{LS} is linear. $x_{e'0}$ and $x_{e'\infty}^*$ are the initial and ultimate mass fractions of erodible fines under the flow velocity v^* ; k_e and α_{er} are material parameters controlling the overall soil erosion sensitivity and the nonlinear variation of $x_{e'\infty}$ with v^{LS} .

MPM SIMULATIONS

The governing equations are discretized spatially by the undeformed Generalized Interpolation Material Point Method (uGIMP, Bardenhagen & Kober, 2004) with 4 material points per cell in the NairnMPM code. These equations are solved explicitly following the order (3)-(4)-(2) within each time step.

Internal erosion tests

The seepage-erosion coupled material point method is validated through the simulation of the internal erosion tests reported in Sterpi (2003). The soil specimen adopted by Sterpi (2003) consists of sand and silt, with the height of 14 cm and the diameter of 7 cm. Its fines content is 23 % and void ratio is 0.51. The erosion tests were performed with different hydraulic gradients, which generated upward seepage flow with different velocities, leading to different erosion rates. The mass of eroded fine particles was measured after drying the water flux from the top of specimens which contains fine particles.

As shown by Figure 2(a), the saturated soil column in the MPM is represented by a 2D model with the same size as the soil specimen. The 2D MPM model is meshed into 7 elements with 4 particles per cell. An additional ghost layer will be added automatically to each boundary in NairnMPM when performing uGIMP. All the boundaries for the solid phase have been fixed; both the top and bottom boundaries are permeable for water, while the lateral boundaries are impermeable. In these simulations, the grid nodes on the bottom boundaries are fixed with 4 different seepage velocities in order to generate the Darcy flows corresponding to the hydraulic gradients of 0.18, 0.39, 0.55 and 0.60.

Table 1 Parameters for erosion tests

k_e	$x_{e'0}$	$x_{e'\infty}^*$	v^* [m/h]	α_{er}
0.250	0.200	0.190	0.234	0.200

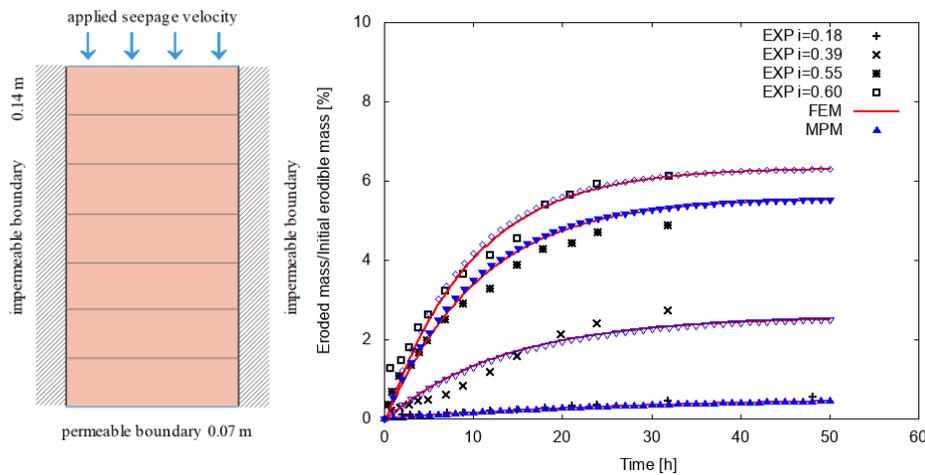


Figure 2 (a) Initial and boundary conditions for MPM model; (b) Evaluation profiles of erodible mass fraction

In this 1D simulation, the hydraulic gradients as well as the velocity fields are homogenous along the depth; Therefore, homogenous erosion rate and fine content will be generated. According to the erosion model, fine particles inside soils will be eroded until the fine content fraction reaches the ultimate fine content fraction which

is dependent on the intrinsic seepage velocity. The MPM simulation replicates the FEM results as well as the experimental results quite well. The discrepancy in the erodible mass, between the numerical and the experimental results is due to the inaccuracies in the ultimate mass fraction curve, as shown in Uzuoka et al. (2012).

Infiltration induced fines migration

This example considers a 1D unsaturated soil column of 1 m length. Similar to previous simulations, the soil column is also represented by a 2D MPM model which is meshed into 50 elements with 4 particles per cell. The initial mass contents of fine particles deposited at the pore surfaces is 340 kg/m^3 . Initially, no fluidized fine particle exists in the pore water ($c_e = 0$), and the concentration of the fluidized particles at the top surface is fixed at zero during the whole infiltration process. The example also assumes that the fines transport is due to advection only. Other circumstances are similar to the one presented in Yerro (2015): the column is assumed to be unsaturated with a constant suction ($s = 0.1 \text{ MPa}$); the bottom boundary is impermeable; gravity is neglected and the soil skeleton cannot deform; the permeability is constant and the water retention curve is linearized according to $S_L = 1 - \alpha_s s$. At time $t = 0+$, the suction at the top of the column is enforced to zero to simulate the water infiltration phenomena. This hydraulic loading induces penetration of water into the unsaturated soil column via the top boundary and subsequent fines migration process.

Table 2 Parameters for infiltration tests

k_e	$x_{e'0}$	$x_{e'\infty}^*$	\mathbf{v}^* [m/s]	α_{er}
0.25	0.20	0.19	6.5e-5	0.20
n	k_L [m/s]	ρ_L [kg/m ³]	$\rho_{e'}$ [kg/m ³]	α_s [Pa ⁻¹]
0.30	5.0e-8	1000	2600	0.000001

Overall, the infiltration induced fine particle migration process simulated here is consistent with the similar case addressed in Lei et al. (2017a): with water infiltration from the top surface, the suction decreases gradually, while the water saturation increase at the meantime; the infiltration generates the downward seepage water flow which erodes fine particles from the soil skeleton with time; accordingly the eroded fines become parts of liquid phase, whose concentration increases with the continuous erosion process, and are transported downwardly by the flowing water.

The numerical results obtained with the uGIMP simulation are close to the ones obtained by FEM with the same number of elements, with some small discrepancies showing up in time of simulation. One reason for these discrepancies (e.g. Figure 3) may be that the MPM simulations presented here have neglected the advective flux due to the pore water saturation gradient term in the water mass balance calculation by assuming that the spatial variation of water is small. This assumption will lead to slower suction dissipation for soils with large α_s value as demonstrated by Yerro (2015). In the explicit uGIMP calculations, the erosion rate is strongly dependent on the resulted seepage flux, while the concentration evolution is dependent on both the advection flux and the eroded mass amount. As the consequence, the discrepancies in suction dissipation profiles will be inherited in the erodible fines mass fraction profiles and the eroded fines concentration profiles.

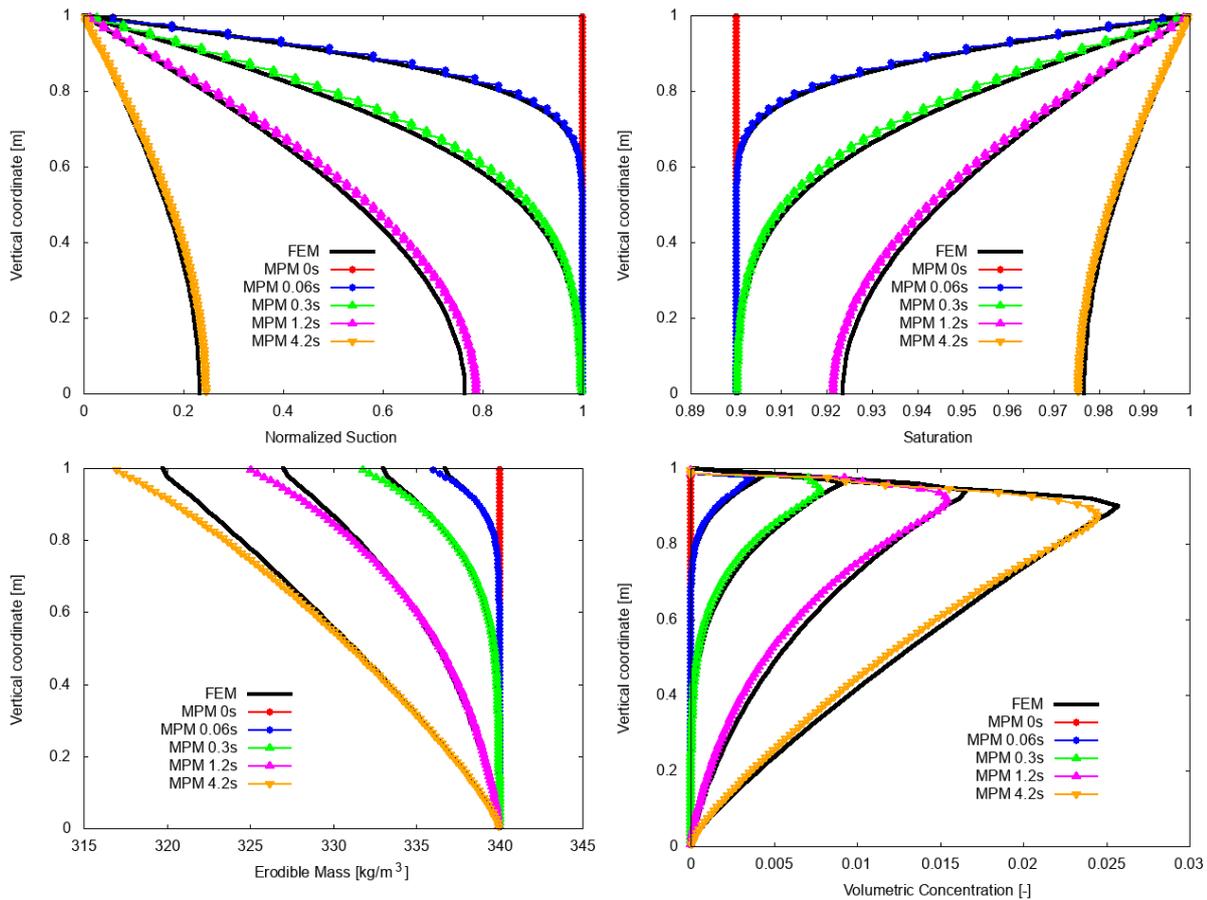


Figure 3 Profiles of suction, saturation, erodible mass fraction, volumetric concentration simulated by FEM and MPM

CONCLUSIONS

This paper proposes a coupled seepage-erosion formulation suitable for the Material Point Method based on the mixture theory. This formulation as well as its implementation into an explicit uGIMP code have been validated firstly by simulating internal erosion tests in saturated soils and then by simulating infiltration induced fines migration process within an unsaturated soil column. We conclude that the proposed coupled seepage-erosion MPM can well capture the main features of the fine particle migration process within unsaturated soils, and its accuracy could be further improved by taking the spatial variation of water content into account.

ACKNOWLEDGEMENTS

This research was funded by the Finland Academy project ‘Progressive failure and post-failure modelling of slopes with Generalized Interpolation Material Point Method (GIMP)’ (286628). The first author would like to acknowledge supports from NSFC (41702331) and CAS (Y7R2070070) for the theoretical development related to fines migration

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